

Some problems to practice:

Make a truth table for these. Show each step:

1. $p \vee (\sim p \wedge q)$
2. $(p \vee r) \wedge \sim (q \vee r)$
3. $\sim p \rightarrow \sim (p \vee q)$
4. $(p \wedge (q \vee r)) \rightarrow ((p \wedge q) \wedge r)$
5. $(p \wedge \sim q) \rightarrow (p \wedge r) \vee \sim (q \wedge r)$

Are any of these statements a tautology?

Show these statements are logically equivalent:

6. $(p \vee (q \wedge r))$ and
 $(p \vee r) \wedge (q \vee r)$

Use deMorgan's laws to rewrite these statements:

7. $\sim (p \vee q)$
8. $\sim p \vee \sim q$

Tell the negation of these statements:

9. The number is prime and odd
10. Every number in set S is either even or a multiple of 5

Show a set diagram for each of the following. Show the steps you need to get the final set diagram. Do any of these describe the same set?

11. $A \cup (B \cap \bar{C})$
12. $\overline{(A \cup B)} \cup C$
13. $A \cup (\overline{B \cup C})$

14. Prove that if a is odd and b is odd, then ab is odd.

For each of the relations described below, decide if it is reflexive, symmetric or transitive. If it is an equivalence relation, tell how many equivalence classes there are.

15. In the integers, xRy if $y - x = 2$ or $x - y = 2$
16. In the integers xRy if $y - x$ is a multiple of 8.

17. In the set $\{1, 2, 3, 4, 5, 6\}$, the relation is given by the ordered pairs:

- (1,1), (1,2), (1,3)
- (2,1), (2,2), (2,3)
- (3,1), (3,2), (3,3)
- (4,4), (4,5)
- (5, 4), (5,5)
- (6,6)

18. In the real numbers, xRy if $x \leq y$

For the equivalence relations below, describe the equivalence class of the given element:

19. In the integers, xRy if $y - x$ is a multiple of 8. Describe the equivalence class containing 6.

20. In the real numbers, xRy if the greatest integer function has the same value for both x and y (the greatest integer function, sometimes called the floor function, is the function that returns the greatest integer that is less than or equal to the number).

Describe the equivalence class containing π .

21. Do the following computations mod 9:

- a. $3 \cdot 8 + 7$
- b. $4 - 6 \cdot 5$
- c. $6^{17} + 5$

22. For each of these functions, decide if it is 1-1 and if it is onto:

- a. $f(x) = x^2 + 7$ on the real numbers
- b. $y = 2^x$ on the real numbers
- c. $f(x) = 2x$ on the integers

23. Prove that these functions are 1-1:

- a. $f(x) = 4x + 2$ on the integers
- b. $f(x) = x^3$ on the real numbers

24. Prove that these functions are onto:

- a. $f(x) = x - 5$ on the integers
- b. $f(x) = x^3 - x$ on the real numbers.

25. Prove that the composition of two 1-1 functions is 1-1.

26. Prove that the composition of two onto functions is onto.