1. $\int \underbrace{x^2}_{u} \underbrace{\sin(3x)dx}_{dv}$

$$\int u\,dv = uv - \int v\,du$$

$du = 2x\,dx \qquad v = \int \sin(3x)dx$

$w = 3x$

$dw = 3dx$

$\frac{1}{3}dw = dx$

$\int \sin w \frac{1}{3}dw = -\frac{\cos w}{3} = \frac{-\cos(3x)}{3}$

$$\frac{-x^2\cos 3x}{3} + \int \frac{\cos(3x)}{3} 2x\,dx$$

$$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\int \underbrace{x}_{u} \underbrace{\cos(3x)dx}_{dv}$$

$du = dx$

$\int \cos(3x)dx$

$v = \frac{1}{3}\int \cos w\,dw = \frac{1}{3}\sin w = \frac{1}{3}\sin(3x)$

$$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} - \int \frac{1}{3}\sin 3x\,dx\right)$$

$$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} + \frac{1}{3}\,\frac{\cos 3x}{3}\right) + C$$

$$-\frac{x^2\cos(3x)}{3} + \frac{2x\sin(3x)}{9} + \frac{2\cos(3x)}{27} + C$$

2. $\int \sin^2 x \cos^3 x \, dx$

$u = \sin x \qquad du = \cos x \, dx$

$$\int \sin^2 x \cos^2 x \cos x \, dx$$

$$\int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$\int u^2 (1 - u^2) \, du$$

$$= \int u^2 - u^4 \, du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

3. $\int \underbrace{\sin^{-1} x}_{u} \, \underbrace{dx}_{dv}$

$\int u \, dv = uv - \int v \, du$

$du = \frac{1}{\sqrt{1 - x^2}} dx$

$dv = dx$

$v = x$

$$= x \sin^{-1} x - \int \frac{x}{\sqrt{1 - x^2}} dx$$

$u = 1 - x^2$

$du = -2x \, dx$

$-\frac{1}{2} du = x \, dx$

$$= x \sin^{-1} x - \int \frac{1}{\sqrt{u}} \left(-\frac{1}{2}\right) du$$

$$= x \sin^{-1} x + \frac{1}{2} \int u^{-1/2} \, du = x \sin^{-1} x + \frac{1}{2} \frac{u^{1/2}}{1/2} + C$$

$$= x \sin^{-1} x + \sqrt{1 - x^2} + C$$

4. $\int \frac{2x^2+3x-8}{(x-4)(x+2)^2}dx$

$$\frac{2x^2+3x-8}{(x-4)(x+2)^2} = \frac{A}{x-4} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$(x-4)(x+2)^2$ $\quad (x-4)(x+2)^2 \quad (x-4)(x+2) \quad (x-4)$

$$2x^2+3x-8 = Ax^2+4Ax+4A+Bx^2-2Bx-8B+Cx-4C$$

$2 = A+B$

$3 = 4A-2B+C \xrightarrow{\times 4}$ $12 = 16A-8B+4C$

$-8 = 4A-8B-4C$ $\qquad -8 = 4A-8B-4C$

$\qquad 4 = 20A-16B$

$\xrightarrow{\times 16}$ $32 = 16A+16B$

$36 = 36A \rightarrow A=1$ :)

$2 = 1+B$ $\qquad 3 = 4-2+C$ $\qquad C=1$

$1 = B$ $\qquad 3 = 1+C$

$$= \int \frac{1}{x-4}dx + \int \frac{1}{x+2}dx + \int \frac{1}{(x+2)^2}dx$$

$du = dx \quad u = x-4 \qquad v = x+2 \quad dv = dx$

$$= \int \frac{1}{u}du + \int \frac{1}{v}dv + \int \frac{1}{v^2}dv$$

$\frac{1}{v^2} = v^{-2}$

$$= \ln|u| + \ln|v| + \frac{v^{-1}}{-1} + C$$

$$= \ln|x-4| + \ln|x+2| - (x+2)^{-1} + C$$

5. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= \int \frac{1}{u}(-1)\,du$

$= -\ln|u| + C = -\ln|\cos x| + C$

$= \ln|\sec x|$

6. $\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$

$\int u\,dv = uv - \int v\,du$

$dv = dx$

$v = x$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x\ln x - \int x\,\frac{1}{x}\,dx$

$x\ln x - \int 1\,dx$

$x\ln x - x + C$

7. $\int \frac{4x^2+5x}{(x-1)(x^2+2)}\,dx$

$$\frac{4x^2+5x}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2}$$

$$4x^2+5x = Ax^2+2A+Bx^2-Bx+Cx-C$$

$$4 = A+B$$
$$5 = -B+C$$
$$0 = 2A-C$$
$$5 = 2A-B$$

$$4 = A+B$$
$$5 = 2A-B$$
$$9 = 3A$$
$$A = 3$$

$$4 = 3+B \rightarrow B = 1$$

$$5 = -1+C$$
$$6 = C$$

$$\int \frac{3}{x-1}\,dx + \int \frac{x+6}{x^2+2}\,dx = \int \frac{3}{x-1}\,dx + \int \frac{x\,dx}{x^2+2} + \int \frac{6\,dx}{x^2+2}$$

$u = x-1$, $du = dx$

$v = x^2+2$, $dv = 2x\,dx$, $\frac{1}{2}dv = x\,dx$

$x^2+2 = 2w^2+2$, $x = \sqrt{2}w$, $dx = \sqrt{2}\,dw$, $w = \frac{x}{\sqrt{2}}$

$$\int \frac{3}{x-1}\,dx + \int \frac{1}{v}\frac{1}{2}\,dv + \int \frac{6}{(\sqrt{2}w)^2+2}\sqrt{2}\,dw$$

$$\int \frac{3}{u}\,du + \frac{1}{2}\ln|v| + \int \frac{6\sqrt{2}}{2w^2+2}\,dw$$

$$= 3\ln|u| + \text{''} + \frac{3\cancel{6}\sqrt{2}}{\cancel{2}}\int \frac{1}{w^2+1}\,dw$$

$$= 3\ln|x-1| + \frac{1}{2}\ln|x^2+2| + 3\sqrt{2}\tan^{-1}(w)$$

$$= 3\ln|x-1| + \frac{1}{2}\ln|x^2+2| + 3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

8. $\int \frac{2}{(9x^2-1)^{3/2}}\,dx$

$\sec^2\theta - 1 = \tan^2\theta$

$\sec^2\theta - \cancel{1} = 9x^2 - \cancel{1}$

$\sec\theta = 3x$

$\frac{\sec\theta}{3} = x$

$\frac{\sec\theta\tan\theta\,d\theta}{3} = dx$

$$\int \frac{2}{\left(9\left(\frac{\sec\theta}{3}\right)^2-1\right)^{3/2}}\,\frac{\sec\theta\tan\theta}{3}\,d\theta$$

$$= \int \frac{2\sec\theta\tan\theta}{3\left(\cancel{9}\,\frac{\sec^2\theta}{\cancel{9}}-1\right)^{3/2}}\,d\theta$$

$$= \int \frac{2\sec\theta\tan\theta}{3(\tan^2\theta)^{3/2}}\,d\theta = \int \frac{2\sec\theta\cancel{\tan\theta}}{3\tan^{\cancel{3}\,2}\theta}\,d\theta$$

$$= \int \frac{2\sec\theta}{3\tan^2\theta}\,d\theta = \int \frac{2}{3}\,\frac{1}{\cancel{\cos\theta}}\,\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \int \frac{2}{3}\,\frac{1}{u^2}\,du = \int \frac{2}{3}u^{-2}\,du = \frac{2}{3}\,\frac{u^{-1}}{-1} + C$$

$\frac{h}{a} = \sec\theta = 3x$


$$= -\frac{2}{3\sin\theta} + C = -\frac{2\cdot\cancel{3}x}{\cancel{3}\sqrt{9x^2-1}} + C = \boxed{\frac{-2x}{\sqrt{9x^2+1}} + C}$$

$$= \frac{-2x\sqrt{9x^2-1}}{9x^2+1} + C$$

10. $\int \frac{x}{e^x}\,dx = \int \underbrace{x}_{u}\,\underbrace{e^{-x}\,dx}_{dv}$

$u = x \qquad du = dx$

$dv = e^{-x}dx \qquad v = \int e^{-x}\,dx \qquad w = -x \qquad dw = -dx$

$v = \int e^{w}(-dw)$

$v = -e^{w} = -e^{-x}$

$= x(-e^{-x}) - \int -e^{-x}\,dx$

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x} + C$

Problem 9 is on page 9 with problem 12

11. $\displaystyle\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

$\tan^2\theta + 1 = \sec^2\theta$

$4\tan^2\theta + 4 = 4\sec^2\theta$

$4\tan^2\theta + \cancel{4} = \cancel{4} + x^2$

$2\tan\theta = x$

$2\sec^2\theta\,d\theta = dx$

$$\int \frac{1}{(2\tan\theta)^2\sqrt{4+(2\tan\theta)^2}}\,2\sec^2\theta\,d\theta$$

$$\int \frac{2\sec^2\theta\,d\theta}{4\tan^2\theta\sqrt{\underbrace{4\quad+4\tan^2\theta}}} \qquad \sqrt{4(1+\tan^2\theta)}$$

$$\int \frac{\cancel{2}\sec^2\theta\,d\theta}{2\,\cancel{4}\tan^2\theta\sqrt{4\sec^2\theta}} = \int \frac{\sec^{\cancel{2}}\theta\,d\theta}{2\tan^2\theta\;2\cancel{\sec\theta}}$$

$$= \frac{1}{4}\int \frac{\sec\theta\quad d\theta}{\tan^2\theta} = \frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\,\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$$= \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

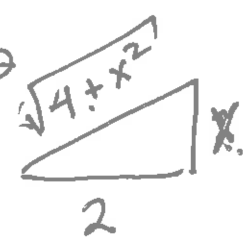
$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}\int u^{-2}\,du = \frac{1}{4}(-1)\,u^{-1} + C$$

$x = 2\tan\theta$

$\tan\theta = \dfrac{x}{2}$




$$= -\frac{1}{4\sin\theta} + C$$

$$= -\frac{\sqrt{4+x^2}}{4x} + C$$

$\sin\theta = \dfrac{x}{\sqrt{4+x^2}}$

9. $\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx = \tan x - x + C$

12. $\int \underbrace{x^3 \underbrace{\ln x}\,dx}$

$u = \ln x \qquad du = \frac{1}{x}\,d\cancel{x} \qquad dv = x^3\,dx$

$v = \int x^3\,dx = \frac{x^4}{4}$

$$= \frac{x^4 \ln x}{4} - \int \frac{\cancel{x^4}^{\,3}}{4} \cdot \frac{1}{\cancel{x}}\,dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3\,dx = \frac{x^4 \ln x}{4} - \frac{1}{4}\frac{x^4}{\cdot 4} + C$$

$$= \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C$$

13. $\int_0^{\pi} \sin^2 x \, dx$

$\cos 2x = 1 - 2\sin^2 x \to \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= \int_0^{\pi} \frac{1}{2}(1 - \cos x)\, dx = \int_0^{\pi} \frac{1}{2}\, dx - \int_0^{\pi} \cos 2x \, dx$

$u = 2x$

$du = 2dx$

$\frac{1}{2} du = dx$

$= \frac{1}{2}x \Big|_0^{\pi} - \int \cos u \, \frac{1}{2} du$

$= \frac{1}{2}x \Big|_0^{\pi} - \frac{\sin u}{2} \Big|$

$= \frac{1}{2}x - \frac{\sin 2x}{2} \Big|_0^{\pi} = \boxed{\frac{1}{2}\pi}$

Alternate

$\frac{1}{2}x \Big|_0^{\pi} - \int_0^{2\pi} \cos u \, \frac{1}{2} du$

$0 \to 2 \cdot 0 = 0$

$\pi \to 2\pi$

$= \frac{1}{2}x \Big|_0^{\pi} - \frac{\sin u}{2} \Big|_0^{2\pi}$

$= \frac{1}{2}\pi - 0 - \left(\frac{\sin 2\pi}{2} - \frac{\sin 0}{2}\right) = \frac{1}{2}\pi$

$$1-\sin^2\theta = \cos^2\theta$$
$$25-25\sin^2\theta = 25\cos^2\theta$$
$$\cancel{25}-x^2 = \cancel{25}-25\sin^2\theta$$
$$x^2 = 25\sin^2\theta$$
$$x = 5\sin\theta$$
$$dx = 5\cos x\, dx$$

14. $\displaystyle\int \frac{3x^2}{\sqrt{25-x^2}}\,dx$

$$= \int \frac{3(5\sin\theta)^2}{\sqrt{25-(5\sin\theta)^2}}\,5\cos\theta\, d\theta$$

$$= \int \frac{3\cdot 25\sin^2\theta}{\sqrt{25-25\sin^2\theta}}\,5\cos\theta\, d\theta$$

$$= \int \frac{75\sin^2\theta}{\sqrt{25(1-\sin^2\theta)}}\,5\cos\theta\, d\theta$$

$$= \int \frac{75\sin^2\theta}{\sqrt{25\cos^2\theta}}\cdot 5\cos\theta\, d\theta = \int \frac{75\sin^2\theta}{\cancel{5\cos\theta}}\cdot \cancel{5\cos\theta}\, d\theta = \int 75\sin^2\theta\, d\theta$$

$$\boxed{\begin{array}{l}\cos 2\theta = 1-2\sin^2\theta \\ \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\end{array}}$$

$$= 75\int \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{75}{2}\int 1\, d\theta - \frac{75}{2}\int \cos 2\theta\, d\theta$$

$u = 2\theta$
$du = 2\,d\theta$
$d\theta = \frac{1}{2}du$

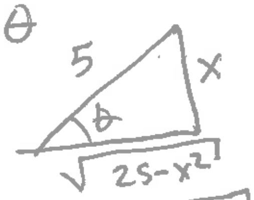
$$= \frac{75}{2}\theta - \frac{75}{2}\int \cos u\cdot\frac{1}{2}du = \frac{75}{2}\theta - \frac{75}{2}\cdot\frac{\sin u}{2} + C$$

$$= \frac{75\theta}{2} - \frac{75}{4}\cdot\sin 2\theta + C = \frac{75\theta}{2} - \frac{75}{\cancel{4}\,2}\,\cancel{2}\sin\theta\cos\theta + C$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$= \frac{75}{2}\sin^{-1}\left(\frac{x}{5}\right) - \frac{\overset{3}{\cancel{75}}}{2}\cdot\frac{x}{\cancel{5}}\,\frac{\sqrt{25-x^2}}{\cancel{5}} + C$$

$x = 5\sin\theta$
$\frac{x}{5} = \sin\theta$
$\sin^{-1}\left(\frac{x}{5}\right) = \theta$




$\cos\theta = \frac{\sqrt{25-x^2}}{5}$

$$= \boxed{\frac{75}{2}\sin^{-1}\left(\frac{x}{5}\right) - \frac{3}{2}x\sqrt{25-x^2} + C}$$

15. $\displaystyle\int \frac{4x^2+3x-1}{2x^2-x-6}\,dx$

$$\begin{array}{r|l} & \quad 2 \\ \hline 2x^2-x-6 & 4x^2+3x-1 \\ & -(4x^2-2x-12) \\ \hline & \quad 5x+11 \end{array}$$

$$= \int 2 + \frac{5x+11}{(2x+3)(x-2)}\,dx$$

$$\cancel{(2x+3)(x-2)}\frac{5x+11}{\cancel{(2x+3)(x-2)}} = \frac{A}{\cancel{2x+3}}(\cancel{2x+3})(x-2) + \frac{B}{\cancel{x-2}}(2x+3)(\cancel{x-2})$$

$$5x+11 = Ax-2A+2Bx+3B$$

$$5 = A+2B \xrightarrow{\times 2} 10 = 2A+4B$$

$$11 = -2A+3B \longrightarrow \begin{array}{r} 11 = -2A+3B \\ \hline 21 = \quad 7B \end{array} \rightarrow B=3$$

$$5 = A+2\cdot 3 \rightarrow A=-1$$

$$\int 2 + \frac{-1}{2x+3} + \frac{3}{x-2}\,dx = \int 2\,dx + \int \frac{-1}{2x+3}\,dx + \int \frac{3}{x-2}\,dx$$

$u = 2x+3$, $du = 2\,dx$, $dx = \frac{1}{2}du$

$v = x-2$, $dv = dx$

$$= \int 2\,dx + \int \frac{-1}{u}\cdot\frac{1}{2}\,du + \int \frac{3}{v}\,dv$$

$$= 2x - \frac{1}{2}\ln|u| + 3\ln|v| + C$$

$$= 2x - \frac{1}{2}\ln|2x+3| + 3\ln|x-2| + C$$

16 $\int_0^{\pi/4} \sec^4 x \, dx = \int_0^{\pi/4} (\tan^2 x + 1)\sec^2 x \, dx$ $\quad u = \tan x$, $du = \sec^2 x \, dx$

Way #1

$$= \int (u^2+1)\, du = \frac{u^3}{3} + u \Big| = \frac{\tan^3 x}{3} + \tan x \Big|_0^{\pi/4}$$

$$= \frac{1}{3} + 1 - (0+0) = \frac{4}{3}$$

Way #2

Alternate substitution $\quad \pi/4 \to \tan \pi/4 = 1$

$0 \to \tan 0 = 0$

$$\int_0^1 (u^2+1)\,du = \frac{u^3}{3} + u \Big|_0^1 = \frac{1}{3} + 1 - (0+0) = \frac{4}{3}$$