Math 167 Final review solutions 2016 (Ch 6)

1. a. $f = 2^x = e^{\ln 2^x} = e^{x\ln 2}$

$\frac{d}{dx}(x\ln 2)$

$$\frac{df}{dx} = \frac{d}{dx}\left(e^{x\cdot\ln 2}\right) = e^{x\ln 2}\cdot\ln 2 = 2^x\ln 2$$

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b. $f = 2^{\tan x} = e^{\ln 2^{\tan x}} = e^{\tan x\cdot\ln 2}$

$$\frac{df}{dx} = e^{\tan x\cdot\ln 2}\underbrace{\left(\sec^2 x\ \ln 2\right)} = 2^{\tan x}\cdot\sec^2 x\cdot\ln 2$$

$\frac{d}{dx}(\tan x\cdot\ln 2)$

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c. $f = x^x = e^{\ln x^x} = e^{x\ln x}$

$$\frac{df}{dx} = e^{x\ln x}\underbrace{\left(1\cdot\ln x + x\cdot\frac{1}{x}\right)}_{\frac{d}{dx}(x\ln x)} = x^x\left(\ln x + 1\right)$$

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d. $f = (\cos x)^x = e^{\ln(\cos x)^x} = e^{x\ln(\cos x)}$

$$\frac{df}{dx} = e^{x\ln(\cos x)}\underbrace{\left(1\cdot\ln(\cos x) + x\cdot\frac{1}{\cos x}(-\sin x)\right)}$$

$\frac{d}{dx}(x\ln(\cos x))$

$$= (\cos x)^x\left(\ln(\cos x) - x\frac{\sin x}{\cos x}\right)$$

2a. $\frac{d}{dx}\left(\sin^{-1}(4x)\right) = \frac{1}{\sqrt{1-(4x)^2}} \cdot 4 = \frac{4}{\sqrt{1-16x^2}}$

b. $\frac{d}{dx} \frac{e^{3x}+\cos x}{x^2} = \frac{(3e^{3x}-\sin x)x^2 - 2x(e^{3x}+\cos x)}{x^4}$

$= \frac{\cancel{x}\left[(3e^{3x}-\sin x)x - 2(e^{3x}+\cos x)\right]}{x^{\cancel{4}\,3}}$

$= \frac{3xe^{3x} - x\sin x - 2e^{3x} - 2\cos x}{x^3}$

not required

c. $\frac{d}{dx} \ln(x^3+3x+1) = \frac{1}{x^3+3x+1}(3x^2+3) = \frac{3x^2+3}{x^3+3x+1}$

d. $\frac{d}{dx} \sqrt{x}\, e^{4x} = \frac{1}{2}x^{-1/2}e^{4x} + \sqrt{x}\, e^{4x} \cdot 4$

$= e^{4x}\left(\frac{1}{2\sqrt{x}} + 4\sqrt{x}\right)$ not required

3.a. $\int \frac{1+3x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{3x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx \rightarrow x\,dx = -\frac{1}{2}\,du$

$= \sin^{-1}x + \int \frac{3}{\sqrt{u}}\left(-\frac{1}{2}\right)du$

$= \sin^{-1}x - \frac{3}{2}\int u^{-1/2}\,du$

$= \sin^{-1}x - \frac{3}{\cancel{2}}\,\frac{u^{1/2}}{\cancel{1/2}} + C = \sin^{-1}x - 3\sqrt{1-x^2} + C$

3b. $\int \frac{2}{2-5x}\,dx$

$u = 2-5x$

$du = -5\,dx$

$dx = -\frac{1}{5}\,du$

$= \int \frac{2}{u}\left(-\frac{1}{5}\right)du = -\frac{2}{5}\ln|u| + C = -\frac{2}{5}\ln|2-5x| + C$

3c. $\int 2^x\,dx = \int e^{\ln 2^x}\,dx = \int e^{x\ln 2}\,dx$

$u = x\ln 2$

$u = \ln 2 \cdot dx \nearrow dx = \frac{1}{\ln 2}\cdot du$

$= \int e^u\left(\frac{1}{\ln 2}\right)du = \frac{1}{\ln 2}\int e^u\,du = \frac{1}{\ln 2}e^u + C$

$= \frac{1}{\ln 2}e^{x\ln 2} + C = \frac{2^x}{\ln 2} + C$