Math 167 Final review Solutions 2016 Ch 7.

4. a. $\int x \cos 4x \, dx$ | $u = x \quad du = dx$ | $dv = \cos 4x \, dx$

$v = \int \cos 4x \, dx \qquad w = 4x, \; dw = 4dx, \; \frac{1}{4}dw = dx$

$= \int \cos w \cdot \frac{1}{4} dw$

$= \frac{1}{4}\sin w = \frac{1}{4}\sin 4x.$

$$= x\frac{\sin 4x}{4} - \int \frac{1}{4}\sin 4x \, dx$$

$w = 4x \quad dw = 4dx \qquad dx = \frac{1}{4}dw$

$$= x\frac{\sin 4x}{4} - \int \frac{1}{4}\sin w \cdot \frac{1}{4} dw$$

$$= x\frac{\sin 4x}{4} - \frac{1}{16}(-\cos w) + C = x\frac{\sin 4x}{4} + \frac{\cos 4x}{16} + C$$

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b. $\int \frac{x^2}{e^x} dx = \int x^2 e^{-x} dx$

$u = x^2 \to du = 2x \, dx$

$dv = e^{-x} \to v = \int e^{-x} dx \qquad w = -x, \; dw = -dx, \; -dw = dx$

$= \int e^w(-dw)$

$= -e^w = -e^{-x}$

$$= x^2(-e^{-x}) - \int (-e^{-x}) \cdot 2x \, dx$$

$$= -x^2 e^{-x} + 2\int x e^{-x} dx$$

$u = x \to du = dx$

$dv = e^{-x} \to v = -e^{-x}$

$$= -x^2 e^{-x} + 2\left(x(-e^{-x}) - \int (-e^{-x}) dx\right)$$

$$= -x^2 e^{-x} - 2xe^{-x} + 2\int e^{-x} dx$$

$$= -x^2 e^{-x} - 2xe^{-x} + 2e^{-x} + C$$

$$= e^{-x}(-x^2 - 2x - 2) + C$$

4 c. $\int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$= x \ln x - \int \frac{x}{x} dx = x \ln x - \int 1 \, dx = x \ln x - x + C$

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d. $\int \tan^{-1}(3x) dx$

$u = \tan^{-1}(3x) \rightarrow du = \frac{1}{1+(3x)^2} \cdot 3$

$dv = dx \quad v = x \qquad = \frac{3}{1+9x^2}$

$= x \tan^{-1}(3x) - \int \frac{3x}{1+9x^2} dx$

$w = 1 + 9x^2$

$dw = 18x \, dx$

$\frac{1}{18} dw = x \, dx$

$= x \tan^{-1}(3x) - \int \frac{3}{w} \cdot \frac{1}{18} dw$

$= x \tan^{-1}(3x) - \frac{3}{18} \ln|w| + C$

$= x \tan^{-1}(3x) - \frac{1}{6} \ln|1 + 9x^2| + C$

$\sin^2 x = 1-\cos^2 x$

4e. $\int \sin^3 x \cos^2 x \, dx = \int \sin x \,(1-\cos^2 x) \cos^2 x \, dx$

$u = \cos x \qquad du = -\sin x \, dx$

$= \int (1-u^2)\, u^2 (-du) = \int (u^2 - u^4)(-1)\, du = -\frac{u^3}{3} + \frac{u^5}{5} + C$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

$\sec^2 x = 1 + \tan^2 x$

4f. $\int \sec^4 x \, dx = \int (1 + \tan^2 x) \sec^2 x \, dx$

$u = \tan x \qquad du = \sec^2 x \, dx$

$= \int 1 + u^2 \, du = u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$

$\tan^2 x = \sec^2 x - 1$

g. $\int \tan^3 x \, dx = \int (\sec^2 x - 1) \tan x \, dx$

$= \int \sec^2 x \tan x \, dx - \int \tan x \, dx$

$u = \tan x$
$du = \sec^2 x dx$

$= \int \tan x \sec^2 x \, dx - \int \frac{\sin x}{\cos x} \, dx$

$w = \cos x$
$dw = -\sin x \, dx$
$-dw = \sin x \, dx$

$= \int u \, du - \int \frac{1}{w} (-dw)$

$= \frac{u^2}{2} + \ln|w| + C$

$= \frac{\tan^2 x}{2} + \ln|\cos x| + C$

note: $\ln|\cos x| = -\ln|\sec x|$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

4h. $\int \cos^2 3x\,dx = \int \frac{1}{2}(1+\cos 6x)\,dx$

$u = 6x$

$du = 6\,dx \rightarrow dx = \frac{1}{6}du$

$$= \int \frac{1}{2}(1+\cos u)\frac{1}{6}du = \frac{1}{12}\int 1+\cos u\,du = \frac{1}{12}(u+\sin u)+C$$

$$= \frac{1}{12}\cdot 6x + \frac{1}{12}\sin(6x)+C = \frac{x}{2}+\frac{1}{12}\sin(6x)+C$$

5a. $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

$\left.\begin{aligned} 1+\tan^2\theta &= \sec^2\theta \\ 4+4\tan^2\theta &= 4+x^2 \end{aligned}\right\}$ find substitution

$2\tan\theta = x$ *

$2\sec^2\theta\,d\theta = dx$

$$= \int \frac{1\cdot 2\sec^2\theta\,d\theta}{(2\tan\theta)^2\sqrt{4+4\tan^2\theta}}$$

$$= \int \frac{2\sec^2\theta}{4\tan^2\theta\sqrt{4(1+\tan^2\theta)}}\,d\theta = \int \frac{2\sec^2\theta}{4\tan^2\theta\cdot 2\sqrt{\sec^2\theta}}\,d\theta = \int \frac{\cancel{2}\sec^{\cancel{2}}\theta}{4\tan^2\theta\cdot\cancel{2}\sec\theta}\,d\theta$$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{\cos^2\theta}{\sin^2\theta\,\cos\theta}\,d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

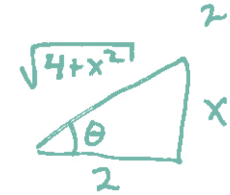
$$= \frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}(-u^{-1})+C = -\frac{1}{4u}+C$$

$$= -\frac{1}{4\sin\theta}+C = -\frac{1}{4\frac{x}{\sqrt{4+x^2}}}+C$$

$$= -\frac{\sqrt{4+x^2}}{4x}+C$$

* $2\tan\theta = x$

$\tan\theta = \frac{x}{2} = \frac{\text{opp}}{\text{adj}}$



note that there are other correct ways to get to the same answer

5b. $\displaystyle\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

$$\cos^2\theta = 1-\sin^2\theta$$
$$9-4x^2 = 9-9\sin^2\theta$$
$$x^2 = \frac{9}{4}\sin^2\theta$$
$$x = \frac{3}{2}\sin\theta^{*} \rightarrow dx = \frac{3}{2}\cos\theta\, d\theta$$

$$= \int \frac{1\cdot 3/2\cos\theta\, d\theta}{\left(\frac{3}{2}\sin\theta\right)^2\sqrt{9-4\left(\frac{3}{2}\sin\theta\right)^2}}$$

$$= \frac{3}{2}\int \frac{\cos\theta\, d\theta}{\frac{9}{4}\sin^2\theta\sqrt{9-9\sin^2\theta}}$$

$$4\cdot\left(\frac{3}{2}\right)^2 = 4\cdot\frac{9}{4} = 9$$

$$= \frac{3}{2}\int \frac{4\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9(1-\sin^2\theta)}} = \frac{3}{2}\cdot\frac{4}{9}\int \frac{\cos\theta\, d\theta}{\sin^2\theta\cdot 3\sqrt{\cos^2\theta}}$$

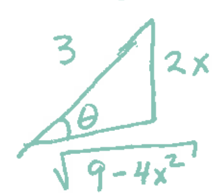
$$= \frac{2}{3}\int \frac{\cancel{\cos\theta}\, d\theta}{3\sin^2\theta\cancel{\cos\theta}} = \frac{2}{3}\cdot\frac{1}{3}\int \csc^2\theta\, d\theta = \frac{2}{9}(-\cot\theta)+C$$

$$= \frac{-2}{9}\,\frac{\sqrt{9-4x^2}}{2x} + C$$

$$= -\frac{\sqrt{9-4x^2}}{9x} + C$$

$*$ $x = \frac{3}{2}\sin\theta$

$\sin\theta = \frac{2x}{3} = \frac{op}{hyp}$



$\cot\theta = \frac{adj}{op} = \frac{\sqrt{9-4x^2}}{2x}$

$$5c.\ \int \frac{x^3}{\sqrt{x^2+25}}\,dx$$

$\tan^2\theta + 1 = \sec^2\theta$

$25\tan^2\theta + 25 = x^2+25$

$25\tan^2\theta = x^2$

$5\tan\theta = x$ ☆

$dx = 5\sec^2\theta\, d\theta$

$$= \int \frac{(5\tan\theta)^3 \cdot 5\sec^2\theta\, d\theta}{\sqrt{(5\tan\theta)^2+25}}$$

$$= \int \frac{125\tan^3\theta\ 5\sec^2\theta\, d\theta}{\sqrt{25\tan^2\theta+25}} = \int \frac{625\tan^3\theta\sec^{2}\theta\, d\theta}{\sqrt{25(\tan^2\theta+1)}}$$

$$= \int \frac{625\tan^3\theta\sec^{2}\theta}{5\sqrt{\sec^2\theta}}\,d\theta = \int \frac{125\tan^3\theta\,\cancel{\sec\theta}^{\,2}}{\cancel{\sec\theta}}\,d\theta$$

$$= 125\int \tan^3\theta\,\cancel{d\theta}\ \sec\theta\, d\theta$$

$\tan^2\theta = \sec^2\theta - 1$

$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$$= 125\int(\sec^2\theta - 1)\sec\theta\tan\theta\, d\theta$$

$$= 125\int(u^2-1)\,du = 125\left(\frac{u^3}{3} - u\right) + C^{\theta}$$

$\cancel{u = \tan\theta}$

$\cancel{du = \sec^2\theta\, d\theta}$

$\cancel{w = \cos\theta}$

$\cancel{dw = -\sin\theta}$

$\cancel{= \frac{u^2}{2} + \ln|w| + C}$

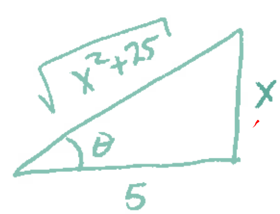
$$= 125\left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C$$

$$= 125\left(\frac{\left(\sqrt{x^2+25}/5\right)^3}{3} - \frac{\sqrt{x^2+25}}{5}\right)$$

$$= 125\left(\frac{(x^2+25)^{3/2}}{75} - \frac{\sqrt{x^2+25}}{5}\right)$$

$5\tan\theta = x$

$\tan\theta = \frac{x}{5} = \frac{op}{adj}$



$\cos\theta = \frac{adj}{hyp} = \frac{5}{\sqrt{x^2+25}}$

$$\cancel{= \frac{125x^2}{2\cdot 25} + \ln 5 - \ln\sqrt{x^2+25} + C}$$

$$\cancel{= \frac{5x^2}{2} + \ln 5 - \frac{1}{2}\ln(x^2+25) + C}$$

not required

5d. $\displaystyle\int \frac{\sqrt{16x^2-1}}{x}\,dx$

$\sec^2\theta - 1 = \tan^2\theta$

$\sec^2\theta - 1 = 16x^2 - 1$

$\sec^2\theta = 16x^2$

$\frac{1}{16}\sec^2\theta = x^2$

$\frac{\sec\theta}{4} = x$ *

$dx = \frac{\sec\theta\tan\theta}{4}$

$$= \int \frac{\sqrt{16\left(\frac{\sec\theta}{4}\right)^2 - 1}\;\frac{\cancel{\sec\theta}\tan\theta}{\cancel{4}}}{\cancel{\sec\theta}/\cancel{4}}\,d\theta$$

$$= \int \sqrt{16\cdot\frac{\sec^2\theta}{16} - 1}\;\tan\theta\,d\theta$$

$$= \int \sqrt{\sec^2\theta - 1}\;\tan\theta\,d\theta = \int \sqrt{\tan^2\theta}\;\tan\theta\,d\theta = \int \tan^2\theta\,d\theta$$

$$= \int \sec^2\theta - 1\,d\theta = \tan\theta - \theta + C$$

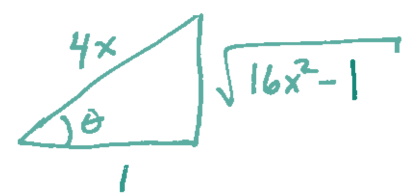
$$= \sqrt{16x^2-1} - \sec^{-1}(4x) + C$$

or

$$= \sqrt{16x^2-1} - \cos^{-1}\left(\frac{1}{4x}\right) + C$$

$\frac{\sec\theta}{4} = x$

$\sec\theta = \frac{4x}{1} = \frac{\text{hyp}}{\text{adj}}$



$\theta = \sec^{-1}(4x)$

or

$\cos\theta = \frac{1}{4x}$

$\theta = \cos^{-1}\left(\frac{1}{4x}\right)$

$\tan\theta = \frac{\text{op}}{\text{adj}} = \sqrt{16x^2-1}$

5e. $\int \frac{1}{3+x^2}\,dx = \int \frac{1}{3(1+x^2/3)}\,dx = \frac{1}{3}\int \frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2}\,dx$

$u = \frac{x}{\sqrt{3}}$ $\quad du = \frac{dx}{\sqrt{3}}$

$\sqrt{3}\,du = dx$

$= \frac{1}{3}\int \frac{1}{1+u^2}\sqrt{3}\,du = \frac{\sqrt{3}}{3}\tan^{-1}(u) + C$

$= \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

alternate:

$\tan^2\theta + 1 = \sec^2\theta$

$3\tan^2\theta + 3 = 3 + x^2$

$\sqrt{3}\tan\theta = x$

$\sqrt{3}\sec^2\theta\,d\theta = dx$

$\int \frac{1}{3+x^2}\,dx = \int \frac{\sqrt{3}\sec^2\theta\,d\theta}{3+3\tan^2\theta} = \frac{\sqrt{3}}{3}\int \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta = \frac{\sqrt{3}}{3}\int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta$

$= \frac{\sqrt{3}}{3}\int 1\,d\theta = \frac{\sqrt{3}}{3}\theta + C$

$\sqrt{3}\tan\theta = x$

$\tan\theta = \frac{x}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$

$= \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

5f. $\displaystyle\int \frac{4x^2+3x-1}{2x^2-x-6}\,dx$

$$\begin{array}{r} 2 \\ 2x^2-x-6\overline{)\,4x^2+3x-1} \\ -(4x^2-2x-12) \\ \hline 5x+11 \end{array}$$

$$= \int 2 + \frac{5x+11}{2x^2-x-6}\,dx = \int 2 + \frac{5x+11}{(2x+3)(x-2)}\,dx$$

$$(2x+3)(x-2)\left(\frac{5x+11}{(2x+3)(x-2)}\right) = \left(\frac{A}{2x+3} + \frac{B}{x-2}\right)(2x+3)(x-2)$$

$$5x+11 = A(x-2) + B(2x+3)$$

$$5x+11 = Ax - 2A + 2Bx + 3B$$

x: $5 = A + 2B \xrightarrow{\times 2} 10 = 2A + 4B$

#: $11 = -2A + 3B \longrightarrow 11 = -2A + 3B$

$21 = 7B$

$3 = B$

$5 = A + 2\cdot 3$

$-1 = A$

$$\int \frac{4x^2+3x-1}{2x^2-x-6}\,dx = \int 2 + \frac{-1}{2x+3} + \frac{3}{x-2}\,dx$$

$$= \int 2\,dx - \int \frac{1}{2x+3}\,dx + 3\int \frac{1}{x-2}\,dx$$

$u = 2x+3$
$du = 2\,dx$
$dx = \frac{1}{2}\,du$

$w = x-2$
$dw = dx$

$$= 2x - \int \frac{1}{u}\left(\frac{1}{2}\right)du + 3\int \frac{1}{w}\,dw$$

$$= 2x - \frac{1}{2}\ln|u| + 3\ln|w|$$

$$= 2x - \frac{1}{2}\ln|2x+3| + 3\ln|x-2| + C$$

5g. $\int \frac{2x^2+3x-8}{(x-4)(x+2)^2}\,dx$

$$(x-4)(x+2)^2\left(\frac{2x^2+3x-8}{(x-4)(x+2)^2}\right) = \left(\frac{A}{x-4}+\frac{B}{x+2}+\frac{C}{(x+2)^2}\right)(x-4)(x+2)^2$$

$$2x^2+3x-8 = A\underbrace{(x+2)^2}_{x^2+2x+2x+4} + B\underbrace{(x-4)(x+2)}_{x^2-4x+2x-8} + C(x-4)$$

$$2x^2+3x-8 = Ax^2+4Ax+4A+Bx^2-2Bx-8B+Cx-4C$$

$2 = A+B$

$3 = 4A-2B+C \rightarrow \times 4 \rightarrow$

$-8 = 4A-8B-4C$

$$12 = 16A-8B+4C$$
$$-8 = 4A-8B-4C$$
$$4 = 20A-16B$$

$\times 16$

$4 = 20A-16B$

$32 = 16A+16B$

$36 = 36A$

$A=1 \rightarrow 2 = 1+B \rightarrow B=1$

$3 = 4\cdot 1 - 2\cdot 1 + C \rightarrow C=1$

$$\int \frac{2x^2+3x-8}{(x-4)(x+2)^2}\,dx = \int \frac{1}{x-4}+\frac{1}{x+2}+\frac{1}{(x+2)^2}\,dx$$

$$= \int \frac{1}{x-4}\,dx + \int \frac{1}{x+2}+\frac{1}{(x+2)^2}\,dx$$

$$= \int \frac{1}{u}\,du + \int \frac{1}{w}+\frac{1}{w^2}\,dw$$

$$= \ln|u| + \ln|w| + \frac{w^{-1}}{-1} + C$$

$$= \ln|x-4| + \ln|x+2| - \frac{1}{x+2} + C$$

$u = x-4$
$du = dx$
$w = x+2$
$dw = dx$

5h. $\int \frac{4x^2+5x}{(x-1)(x^2+2)}\,dx$

$$(x-1)(x^2+2)\left(\frac{4x^2+5x}{(x-1)(x^2+2)}\right)=\left(\frac{A}{x-1}+\frac{Bx+C}{x^2+2}\right)(x-1)(x^2+2)$$

$$4x^2+5x = A(x^2+2)+(Bx+C)(x-1)$$

$$4x^2+5x = Ax^2+2A+Bx^2-Bx+Cx-C$$

$4=A+B$ → $4=A+B$

$5=-B+C$ → $5=-B+C$

$9=A+C$

$0=2A-C$ → $0=2A-C$

$9=3A$

$4=3+B$ ← $\boxed{A=3}$

$B=1$

$5=-1+C \to C=6$

$$\int \frac{4x^2+5x}{(x-1)(x^2+2)}\,dx=\int \frac{3}{x-1}+\frac{x+6}{x^2+2}\,dx$$

$u=x-1$
$du=dx$

$w=x^2+2$
$dw=2x\,dx$
$\frac{1}{2}dw=dx$

$$=\int \frac{3}{x-1}\,dx+\int \frac{x}{x^2+2}\,dx+\int \frac{6}{x^2+2}\,dx$$

$$=\int \frac{3}{u}\,du+\int \frac{1}{w}\frac{1}{2}\,dw+\frac{6}{2}\int \frac{1}{r^2+1}\sqrt{2}\,dr$$

$$=3\ln|u|+\frac{1}{2}\ln|w|+3\sqrt{2}\tan^{-1}(r)+C$$

$$=3\ln|x-1|+\frac{1}{2}\ln|x^2+2|+3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$$

$$\int \frac{6}{x^2+2}\,dx$$

$$=\int \frac{6}{2\left(\frac{x^2}{2}+1\right)}\,dx$$

$$=\frac{6}{2}\int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\,dx$$

$r=\frac{x}{\sqrt{2}}$, $dr=\frac{dx}{\sqrt{2}}$

$\sqrt{2}\,dr=dx$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

6.a. $\displaystyle\int_0^{\pi/3} \sin^2 3x\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{2}(1-\cos 6x)\,dx$

$u = 6x$

$du = 6\,dx \rightarrow dx = \frac{1}{6}du$

$x = 0 \rightarrow u = 6\cdot 0 = 0$

$x = \frac{\pi}{3} \rightarrow u = 6\cdot\frac{\pi}{3} = 2\pi$

$$= \int_0^{2\pi} \frac{1}{2}(1-\cos u)\frac{1}{6}\,du$$

$$= \frac{1}{12}(u - \sin u)\Big|_0^{2\pi} = \frac{1}{12}(2\pi - 0) - \frac{1}{12}(0-0) = \frac{\pi}{6}$$

b. $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx$

$\sec^2\theta - 1 = \tan\theta$

$4\sec^2\theta - 4 = x^2 - 4$

$4\sec^2\theta = x^2$

$2\sec\theta = x$

$dx = 2\sec\theta\tan\theta\,d\theta$

$x = 2 \rightarrow 2\sec\theta = 2 \rightarrow \sec\theta = 1$
$\rightarrow \cos\theta = 1$
$\rightarrow \theta = 0$

$x = 4 \rightarrow 2\sec\theta = 4 = \sec\theta = 2$
$\rightarrow \cos\theta = \frac{1}{2}$
$\rightarrow \theta = \pi/3$

$$= \int_0^{\pi/3} \frac{\sqrt{4\sec^2\theta - 4}\;2\sec\theta\tan\theta\,d\theta}{(2\sec\theta)^4}$$

$$= \int_0^{\pi/3} \frac{\cancel{2}\sqrt{\sec^2\theta - 1}\cdot 2\sec\theta\tan\theta\,d\theta}{{}_4\cancel{16}\sec^4\theta}$$

$$= \int_0^{\pi/3} \frac{\tan\theta\sec\theta\tan\theta\,d\theta}{4\sec^4\theta}$$

$$= \int_0^{\pi/3} \frac{\tan^2\theta}{4\sec^3\theta}\,d\theta = \int_0^{\frac{\pi}{3}} \frac{\sin^2\theta}{4\cos^2\theta}\cdot\frac{\cos^3\theta}{1}\,d\theta$$

$$= \int_0^{\pi/3} \frac{\sin^2\theta\cos\theta\,d\theta}{4}$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\theta = 0 \rightarrow u = \sin 0 = 0$

$\theta = \pi/3 \rightarrow u = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$= \frac{1}{4}\int_0^{\sqrt{3}/2} u^2\,du = \frac{u^3}{4\cdot 3}\Big|_0^{\sqrt{3}/2} = \frac{(\sqrt{3}/2)^3}{4\cdot 3} - 0 = \frac{\sqrt{3}}{8\cdot 4} = \frac{\sqrt{3}}{32}$$

6c. $\int_0^3 2^x\,dx = \int_0^3 e^{\ln 2^x}\,dx = \int_0^3 e^{x\ln 2}$

$= \int_0^{3\ln 2} e^u \cdot \frac{1}{\ln 2}\,du$

$u = x\ln 2 \to du = \ln 2 \cdot dx$

$dx = \frac{1}{\ln 2}\,du$

$= \left.\frac{e^u}{\ln 2}\right|_0^{\ln 8}$

$x = 0 \to u = 0\cdot\ln 2 = 0$

$x = 3 \to u = 3\ln 2 = \ln 2^3 = \ln 8$

$= \frac{e^{\ln 8}}{\ln 2} - \frac{e^0}{\ln 2} = \frac{8 \quad -1}{\ln 2} = \frac{7}{\ln 2}$

6 d. $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx = \int_0^{\pi/2} \sin x\,(1-\cos^2 x)\cos^2 x\,dx$

$u = \cos x$ $\quad \sin x\,dx = -du$

$du = -\sin x\,dx$

$x = 0 \to u = \cos(0) = 1$

$x = \pi/2 \quad u = \cos(\pi/2) = 0$

$= \int_1^0 (1-u^2)u^2(-du)$

$= \int_1^0 -u^2 + u^4\,du = \left.-\frac{u^3}{3} + \frac{u^5}{5}\right|_1^0$

$= 0 - \left(-\frac{1}{3} + \frac{1}{5}\right) = \frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$