Final exam review 2016, probs from Ch 11

7. a. $f = \frac{3}{2+5x} = \frac{3}{2(1+\frac{5x}{2})} = \frac{3}{2}\frac{1}{(1-(\frac{-5x}{2}))}$

$= \frac{3}{2}\sum_{n=0}^{\infty}\left(\frac{-5x}{2}\right)^n = \frac{3}{2}\sum_{n=0}^{\infty}\frac{(-5)^n x^n}{2^n}$ ← correct

$= \sum_{n=0}^{\infty}\frac{3(-1)^n 5^n x^n}{2^{n+1}}$ ← optional

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b. $f = \frac{1}{2+x} = \frac{1}{2(1+\frac{x}{2})} = \frac{1}{2(1-(\frac{-x}{2}))}$

$= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{-x}{2}\right)^n = \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^n} = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^{n+1}}$

↑ correct ↑ optional

c. $\frac{d}{dx}\left(\frac{1}{2+x}\right) = \frac{-1}{(2+x)^2}$

from #b

$\frac{1}{(2+x)^2} = -\frac{d}{dx}\left(\frac{1}{2+x}\right) = -\frac{d}{dx}\overbrace{\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^{n+1}}}$ =

$= -1\sum_{n=0}^{\infty}\frac{(-1)^n n x^{n-1}}{2^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1} n x^{n-1}}{2^{n+1}}$

↑ correct ↑ simplified

$u = 2+x,\ du = dx$

7d. $\int \frac{1}{2+x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|2+x| + C$

$\ln|2+x| = \underbrace{\int \frac{1}{2+x}\,dx}_{\text{from b}} \quad (+C')$

$= \int \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{n+1}}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^{n+1}(n+1)} + C$

$\ln|2+0| = \sum_{n=0}^{\infty} 0 + C \Rightarrow C = \ln 2$

$\ln|2+x| = \ln 2 + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^{n+1}(n+1)}$

8a. $\ln(1.5) = \ln(1+.5) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} .5^n}{n}$ ← plug $.5 = x$ into $\ln(1+x)$ formula

$\approx \sum_{n=1}^{5} \frac{(-1)^{n-1} .5^n}{n}$

$= \frac{.5}{1} - \frac{.5^2}{2} + \frac{.5^3}{3} - \frac{.5^4}{4} + \frac{.5^5}{5} \approx .4073$

8 b. $e^3 = \sum_{n=0}^{\infty} \frac{3^n}{n!} \approx \sum_{n=0}^{4} \frac{3^n}{n!} = \frac{1}{1} + \frac{3}{1} + \frac{3^2}{2} + \frac{3^3}{3!} + \frac{3^4}{4!}$

$= 16.375$

9.a. $\sin x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$

$\int \sin(x^2)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\,dx$

$= \sum_{n=1}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)!(4n+3)} + C$

b. $\cos\sqrt{x} = \sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

$\int \cos\sqrt{x}\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(2n)!(n+1)} + C$

10 a. $\sin(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$

$$\int_0^1 \sin(x^2)\,dx = \int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)!\,(4n+3)} \Big|_0^1$$

$$\approx \sum_{n=0}^{3} \frac{(-1)^n x^{4n+3}}{(2n+1)!\,(4n+3)} \Big|_0^1 = \sum_{n=0}^{3} \frac{(-1)^n 1^{4n+3}}{(2n+1)!\,(4n+3)} - \sum_{n=0}^{3} 0$$

$$= \frac{1}{1!\,3} - \frac{1}{3!\,7} + \frac{1}{5!\,11} - \frac{1}{7!\cdot \cancel{13}\ 15} - 0$$

$$\approx .3103$$

10b. $\ln(1+x^3) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x^3)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n}}{n}$

$$\int_0^{.5} \ln(1+x^3)\,dx = \int_0^{.5} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n}}{n}\,dx =$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n+1}}{n(3n+1)} \Bigg|_0^{.5} \approx \sum_{n=1}^{4} \frac{(-1)^{n-1} x^{3n+1}}{n(3n+1)} \Bigg|_0^{.5}$$

$$= \sum_{n=1}^{4} \frac{(-1)^{n-1} (.5)^{3n+1}}{n(3n+1)} - \sum_{n=1}^{4} \frac{(-1)^{n-1} 0^{3n+1}}{n(3n+1)}$$

$$= \frac{(.5)^4}{1 \cdot 4} - \frac{(.5)^7}{2 \cdot 7} + \frac{(.5)^{10}}{3 \cdot 10} - \frac{(.5)^{13}}{4 \cdot 13} - 0$$

$$\approx .0151$$