11. a. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n^2 4^n}$

$$\lim_{n\to\infty} \frac{x^{2(n+1)}/(n+1)^2 4^{n+1}}{x^{2n}/n^2 4^n} = \lim_{n\to\infty} \frac{x^{2(n+1)} n^2 4^n}{(n+1)^2 4^{n+1} x^{2n}}$$

$$= \lim_{n\to\infty} \frac{x^{2n+2} n^2 4^n}{x^{2n} (n+1)^2 4^{n+1}} = \lim_{n\to\infty} x^2 \left(\frac{n \cdot \frac{1}{n}}{(n+1)\frac{1}{n}}\right)^2 \cdot \frac{1}{4}$$

$$= \lim_{n\to\infty} \frac{x^2}{4}\left(\frac{1}{1+\frac{1}{n}}\right)^2 = \frac{x^2}{4}$$

$$\left|\frac{x^2}{4}\right| < 1 \Rightarrow |x^2| < 4 \Rightarrow -2 < x < 2$$

$x=2$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n}}{n^2 \cdot 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{n^2 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$

$x=-2$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n (-2)^{2n}}{n^2 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{n^2 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$

$\sum \frac{1}{n^2}$ is a p-series that converges

so $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$ converges

and the interval of convergence is $[-2, 2]$

11 b. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n 4^n}$

$$\lim_{n\to\infty} \frac{(x-2)^{n+1}/(n+1)4^{n+1}}{(x-2)^n/n4^n} = \lim_{n\to\infty} \frac{(x-2)^{n+1}\ n\ \ 4^n}{(x-2)^n\ (n+1)\ 4^{n+1}}$$

$$= \lim_{n\to\infty} (x-2)\left(\frac{n\ \frac{1}{n}}{(n+1)\frac{1}{n}}\right)\frac{1}{4}$$

$$= \lim_{n\to\infty} \frac{x-2}{4}\left(\frac{1}{1+\frac{1}{n}}\right) = \frac{x-2}{4}$$

$$-1 < \frac{x-2}{4} < 1$$

$$-4 < x-2 < 4$$
$$+2 \quad\ +2 \quad +2$$

$$-2 < x < 6$$

$x=-2$: $\sum_{n=1}^{\infty} \frac{(-2-2)^n}{n\cdot 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{n 4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n}$

alternating, $\frac{1}{n}$ is decreasing

$\lim_{n\to\infty} \frac{1}{n} = 0$ so it converges

$x=6$: $\sum_{n=1}^{\infty} \frac{(6-2)^n}{n 4^n} = \sum_{n=1}^{\infty} \frac{4^n}{n 4^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

is the harmonic series, which diverges

interval $[-2, 6)$

11 c. $\sum_{n=0}^{\infty} \frac{x^{2n}}{9^n n!}$

$$\lim_{n\to\infty} \frac{x^{2(n+1)}/9^{n+1}(n+1)!}{x^{2n}/9^n n!} = \lim_{n\to\infty} \frac{x^{2n+2}\, 9^n}{x^{2n}\, 9^{n+1}} \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} \frac{x^2}{9(n+1)} = \frac{x^2}{9} \cdot 0 = 0$$

$|x \cdot 0| < 1$ for all values of x

so the interval is $(-\infty, \infty)$

11 d. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$

$$\lim_{n\to\infty} \frac{(x+1)^{2(n+1)}/9^{n+1}}{(x+1)^{2n}/9^n} = \lim_{n\to\infty} \frac{(x+1)^{2n+2}\,9^n}{(x+1)^{2n}\,9^{n+1}}$$

$$= \lim_{n\to\infty} \frac{(x+1)^2}{9}$$

$$\left|\frac{(x+1)^2}{9}\right| < 1$$

$$|(x+1)^2| < 9$$

$$|x+1| < 3$$

$$\begin{aligned} -3 &< x+1 < 3 \\ -1 & \quad\;\; -1 \quad\;\; -1 \end{aligned}$$

$$-4 < x < 2$$

$x=2$: $\displaystyle\sum_{n=0}^{\infty} \frac{(2+1)^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{3^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{9^n}{9^n} = \sum_{n=0}^{\infty} 1$

diverges

$\left(\lim_{n\to\infty} 1 = 1 \neq 0\right)$

$x=-4$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-4+1)^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{(-3)^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{9^n}{9^n}$

diverges

interval: $(-4, 2)$

12. a. $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1}$ alternates

$\frac{1}{3x+1}$ is decreasing

$\lim_{n\to\infty} \frac{1}{3n+1} = 0$

so it converges by the Alternating Series test

12.b. $\sum_{n=1}^{\infty} \frac{n}{4^n}$

$$\lim_{n\to\infty} \frac{(n+1)/4^{n+1}}{n/4^n} = \lim_{n\to\infty} \frac{(n+1)}{n} \frac{4^n}{4^{n+1}} =$$

$$= \lim_{n\to\infty} \left(\frac{(n+1)\frac{1}{n}}{n\ \frac{1}{n}} \right) \cdot \frac{1}{4} = \lim_{n\to\infty} \left(\frac{1+\frac{1}{n}}{1} \right) \cdot \frac{1}{4} = \frac{1}{4}$$

$\left|\frac{1}{4}\right| < 1$ so it converges by the Ratio test.

12.c. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\int_1^{\infty} \frac{1}{x^2}\, dx = \lim_{t\to\infty} -\frac{1}{x}\Big|_1^t$$

$$= \lim_{t\to\infty} -\frac{1}{t} - \left(-\frac{1}{1}\right) = 0 + 1 = 1 \quad (\text{not infinite})$$

converges by integral test (with sum ~ 1)

12.d. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+5}{n^2+3}$ (similar to $\frac{1}{n}$ which diverges)

$$\frac{2n+1}{n^2+3} \geq \frac{2n}{n^2+3} \geq \frac{2n}{n^2+3n^2} = \frac{2n}{4n^2} = \frac{1}{2}\cdot\frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{2}\cdot\frac{1}{n} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$$ diverges (harmonic series)

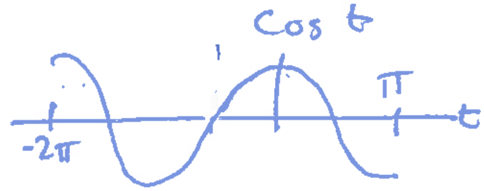
so $\displaystyle\sum_{n=1}^{\infty} \frac{2n+5}{n^2+3}$ diverges by the comparison test.

13 a. $x = t^2 + \pi t$ $\quad -2\pi < t < \pi$

$y = 2\cos t$

$x = t^2 + \pi t$

$t(t+\pi)$

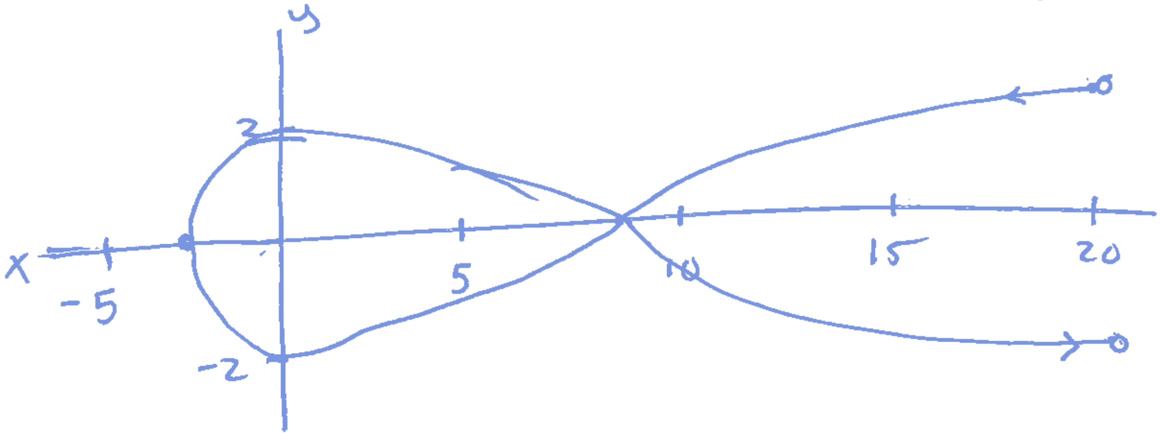


t	-2π	$-3\pi/2$	$-\pi$	$-\pi/2$	0	$\pi/2$	π
x	19.7	7.4	0	-2.5	0	7.4	19.7
y	2	0	-2	0	2	0	-2

note $2\pi^2 \approx 19.7$

$\frac{3\pi^2}{4} \approx 7.4$

$-\frac{\pi^2}{4} \approx -2.5$



14.a. $\frac{dy}{dt} = -2\sin t \qquad \frac{dx}{dt} = 2t + \pi \qquad \frac{dy}{dx} = \frac{-2\sin t}{2t+\pi}$

$t = \pi/2$: $x = \frac{\pi^2}{4} + \frac{\pi^2}{2} = \frac{3\pi^2}{4} \qquad y = 2\cos\frac{\pi}{2} = 0$

$m = \frac{-2\sin \pi/2}{2(\pi/2)+\pi} = \frac{-2 \cdot 1}{\pi + \pi} = \frac{-2}{2\pi} = -\frac{1}{\pi}$

Line : $y - 0 = -\frac{1}{\pi}\left(x - \frac{3\pi^2}{4}\right) \rightarrow y = -\frac{1}{\pi}x + \frac{3\pi}{4}$

15. a. $\frac{dy}{dt} = -2\sin t = 0 \Rightarrow t = -2\pi, -\pi, 0, \pi$

horizontal tangent at $(2\pi^2, 2)^*$, $(0,-2)$, $(0,2)$

$(2\pi^2, -2)^*$

$\frac{dx}{dt} = 2t + \pi = 0$

$2t = -\pi$

$t = -\pi/2$ $\quad$ vertical tangent at $\left(-\frac{\pi^2}{4}, 0\right)$

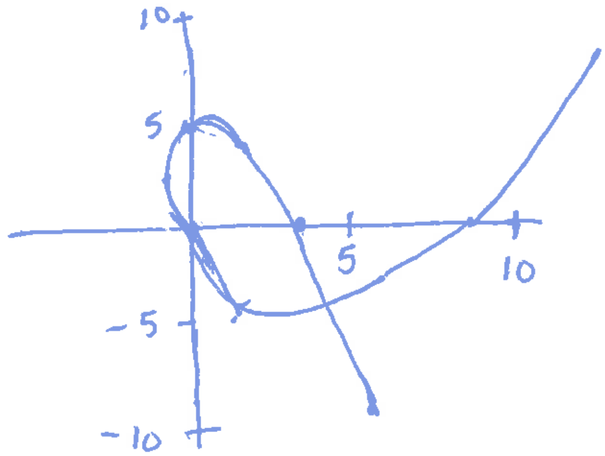
* OK to omit these because the given interval is $-2\pi < t < \pi$

13. b. $x = t^2 + t$
$y = t^3 - 6t$

$t^2 + t = 0$
$t(t+1) = 0$
$t = 0, -1$

$t^3 - 6t = 0$
$t(t^2 - 6) = 0$
$t = 0, \pm\sqrt{6}$

t	-3	$-\sqrt{6}$	-2	-1	$-\frac{1}{2}$	0	1	2	$\sqrt{6}$	3
x	6	3.6	2	0	$-\frac{1}{4}$	0	1	6	8.4	12
y	-9	0	4	5	2.9	0	-5	-4	0	9



15. b.
$\frac{dx}{dt} = 2t + 1 = 0$
$t = -1/2$
$\left(-\frac{1}{4}, \frac{-1}{8} + 3\right)$
$= \left(-\frac{1}{4}, 2.875\right)$
vertical tangent at:

14. b. $(0, 5)$ $x = 0$: $t = 0, -1$
$y = 0$, $y = 5$

$t = -1$

$\frac{dy}{dt} = 3t^2 - 6$ $\frac{dx}{dt} = 2t + 1$ $\frac{dy}{dx} = \frac{3t^2 - 6}{2t + 1}$

$m = \frac{3(-1)^2 - 6}{2(-1) + 1} = \frac{-3}{-1} = 3$

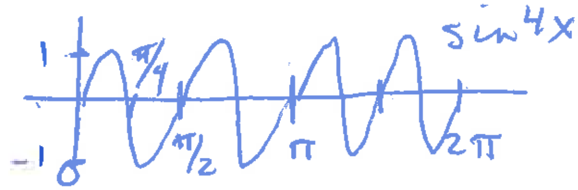
$y - 5 = 3(x - 0) \rightarrow \boxed{y = 3x + 5}$

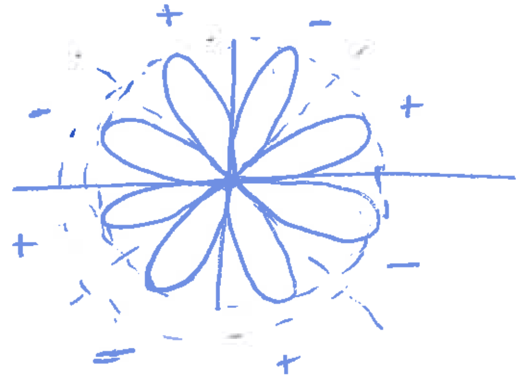
15. b $\frac{dy}{dt} = 3t^2 - 6 = 0$
$3(t^2 - 2) = 0$
$t^2 = 2$
$t = \pm\sqrt{2}$

horizontal tangents at:
$(\sqrt{2}^2 - \sqrt{2}, -\sqrt{2}^3 + 6\sqrt{2})$
$\approx (.586, 5.657)$*
and $(\sqrt{2}^2 + \sqrt{2}, \sqrt{2}^3 - 6\sqrt{2})$
$\approx (3.414, -5.657)$*

*OK to give approximation here

13c. $r = \sin 4\theta$





17a

$$\frac{1}{2}\int_0^{\pi/4} (\sin 4\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{1}{2}(1 - \cos 8\theta)\, d\theta$$

$u = 8\theta$

$du = 8\, d\theta$

$d\theta = \frac{1}{8} du$

$$= \frac{1}{4}\int_0^{2\pi} (1 - \cos u) \cdot \frac{1}{8} du$$

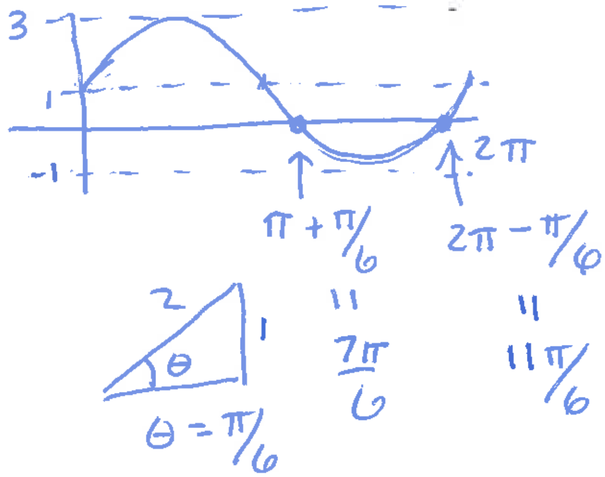
$\theta = 0 \to u = 0$

$\theta = \frac{\pi}{4} \to u = 8 \cdot \frac{\pi}{4} = 2\pi$

$$= \frac{1}{32}(u - \sin u)\Big|_0^{2\pi}$$

$$= \frac{1}{32}\Big[(2\pi - 0) - (0 - 0)\Big] = \frac{2\pi}{32} = \frac{\pi}{16}$$

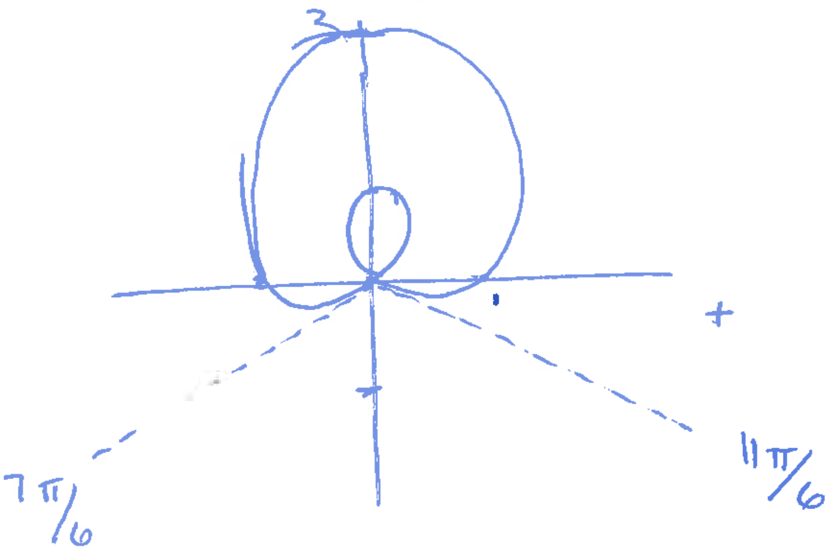
13 d. $r = 2\sin\theta + 1$



$$2\sin\theta + 1 = 0$$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$



17 b. $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (2\sin\theta + 1)^2 \, d\theta = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} 4\sin^2\theta + 4\sin\theta + 1 \, d\theta$

$$= \frac{1}{2}\int_{-\pi/6}^{7\pi/6} 4 \cdot \frac{1}{2}(1 - \cos 2\theta) + 4\sin\theta + 1 \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/6}^{7\pi/6} -2\cos 2\theta + 4\sin\theta + 3 \, d\theta$$

$$= \frac{1}{2}\left(-\sin 2\theta - 4\cos\theta + 3\theta\right)\Big|_{-\pi/6}^{7\pi/6}$$

$$= \frac{1}{2}\left(-\sin\frac{7\pi}{3} - 4\cos\frac{7\pi}{6} + \frac{7\pi}{2}\right) - \frac{1}{2}\left(-\sin\left(-\frac{\pi}{3}\right) - 4\cos\left(-\frac{\pi}{6}\right) - \frac{\pi}{2}\right)$$

$$= \frac{1}{2}\left(-\frac{\sqrt{3}}{2} + \frac{4\sqrt{3}}{2} + \frac{7\pi}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{4\sqrt{3}}{2} - \frac{\pi}{2}\right) = \frac{1}{2}\left(\frac{6\sqrt{3}}{2} + \frac{8\pi}{2}\right) \approx 8.88$$

16. a.

$x = t^2 + \pi t \qquad\qquad y = 2\cos t$

$\frac{dx}{dt} = 2t + \pi \qquad\qquad \frac{dy}{dt} = -2\sin t$

$$\frac{dy}{dx} = \frac{-2\sin t}{2t+\pi}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-2\cos t(2t+\pi) - 2(-2\sin t)}{(2t+\pi)^2}$$

$$= \frac{-4t\cos t - 2\pi\cos t + 4\sin t}{(2t+\pi)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-4t\cos t - 2\pi\cos t + 4\sin t}{(2t+\pi)^2} \cdot \frac{1}{2t+\pi}$$

$$= \frac{-4t\cos t - 2\pi\cos t + 4\sin t}{(2t+\pi)^3}$$

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b. $x = t^2 + t \qquad y = t^3 - 6t \qquad \frac{dy}{dx} = \frac{3t^2-6}{2t+1}$

$\frac{dx}{dt} = 2t+1 \qquad \frac{dy}{dt} = 3t^2 - 6$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(6t)(2t+1) - (3t^2-6)\cdot 2}{(2t+1)^2} = \frac{12t^2+6t-6t^2+12}{(2t+1)^2}$$

$$= \frac{6t^2+6t+12}{(2t+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{6t^2+6t+12}{(2t+1)^2} \cdot \frac{1}{2t+1} = \frac{6t^2+6t+12}{(2t+1)^3}$$