

Derivative of the natural logarithm:

Use the derivative definition:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

and apply it to the function: $f(x) = \ln(x)$

$$\lim_{h \rightarrow 0} \frac{\ln(\quad) - \ln(\quad)}{h} = \lim_{h \rightarrow 0} \frac{\ln\left(\frac{\quad}{\quad}\right)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \cdot \ln\left(\frac{\quad}{\quad}\right) =$$

here write the formula

here combine the logs

here write 1/h as a factor.

$$\lim_{h \rightarrow 0} \ln\left(\left(\frac{\quad}{\quad}\right)\right) = \lim_{h \rightarrow 0} \ln\left(\left(\frac{\quad}{\quad} + \frac{\quad}{\quad}\right)\right) = \lim_{h \rightarrow 0} \ln\left(\left(\frac{\quad}{\quad} + \frac{\quad}{\quad}\right)\right)$$

use the product → exponent rule

turn into 2 fractions

simplify x/x

Now, we're going to make a substitution: $\frac{h}{x} = k$

Solve and figure out: $\frac{1}{h} = \frac{1}{xk}$ when $h \rightarrow 0$ (and $x \neq 0$) figure out $k \rightarrow$

$$= \lim_{k \rightarrow 0} \ln\left(\left(1 + \frac{h}{x}\right)\right) = \lim_{k \rightarrow 0} \ln\left(\left(1 + \frac{1}{k}\right)^k\right) = \ln\left(\left(\frac{e}{e}\right)\right)$$

Make the substitution

Separate the x from the k:

Use $e = \lim_{k \rightarrow 0} (1+k)^{1/k}$

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Simplify

So the derivative of the natural logarithm is:

$$\frac{d}{dx} \ln x =$$

Which means $\int dx = \ln x + C$