

Definition of e (Johann Bernoulli):

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x$$

e from compound interest

Finding interest compounded n times in 1 year at an annual rate of r :

$$\left(1 + \frac{r}{n} \right)^n$$

How can I move r to somewhere more convenient?

Substitute:

$$\frac{1}{x} = \frac{r}{n}$$

So:

$$x = \quad \text{and } n =$$

so if $n \rightarrow \infty$ then $x \rightarrow$

Substituting in for n and r , we get

$$\left(1 + \frac{r}{n} \right)^n = \left(1 + \frac{1}{x} \right)^x$$

Fill in exponent

So, finally we get:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n} \right)^n = \lim_{x \rightarrow} \left(1 + \frac{1}{x} \right)^x$$

Another useful form for defining e :

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x$$

Substitute $h = \frac{1}{x}$, so $x =$

and if $x \rightarrow \infty$ then $h \rightarrow$

Making those substitutions, we get the form:

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = \lim_{h \rightarrow} (1 + h)^{\square}$$

So we can use

$$e = \lim_{h \rightarrow} (1 + h)$$

Definition/formula of derivative:

Geometrically a derivative is the slope of a tangent line.

The function is $f(x)$. We want the derivative at the value x . The number $x + h$ is h distance from x .

The point at x is $(x, f(x))$ The point at $x + h$ is: (\quad , \quad)

The slope between these two points is: $\frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad}$

If we want the slope of a tangent line, then we want the distance between the x -coordinates very small. That distance is h , so we use the limit:

$$\lim_{h \rightarrow 0} \frac{\quad}{\quad}$$

The derivative of e^x

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{\quad}{h} = \lim_{h \rightarrow 0} \frac{\quad}{h} =$$

factor using exponent rules
Substitute for one of the e's using the defn