Info about the chapter 11 test

I. Expect 2-3 problems where you come up with an infinite series expansion for a function
II. Expect 1-2 problems where you find an estimate for a function value, integral or infinite series
III. Expect 1-2 problems where you find a radius of convergence or interpret a radius of convergence
IV. Expect 2-3 problems where you show whether an infinite series converges or diverges using one of the tests

More details:

I. Infinite series for a function problems could be one of the following types:

A. Find an infinite series using the geometric series formula: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ (for $|x|<1$). *Memorize this!*

Sample probs (find a power series expansion for each):

1. $f(x) = \frac{1}{(3+x)^2}$
2. $f(x) = \ln(4+x)$

B. Find an infinite series using Taylor's formula:

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$ *Memorize this!*

Sample probs: Find the first 4 non-zero terms of the Taylor series expansion for each function, centered at a

3. $f(x) = \ln|x| \quad a = 3$
4. $f(x) = \sqrt{x} \quad a = 25$
5. $f(x) = \cos 2x \quad a = \pi$

C. Find a series by using a formula that is given to you
Sample problems:

6. Find the McLaurin series for $f(x) = e^{x^2}$ using $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
7. Find the McLaurin series for $f(x) = x\tan^{-1}(4x)$ using $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

D. Find the series to represent an indefinite integral
Sample problem:

8. Find the McLaurin series for $\int \sin(x^2)dx$ given $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
9. Find the McLaurin series for $\int \cos\sqrt{x}\, dx$ given $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

II. Estimate using a series problems will be of one of the following types:

A. Estimate a function value using a series:
Sample probs. Do all calculations to at least 4 digits.

9. Estimate ln(1.5) using the first 5 terms of $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$. Compare to the calculator value, and find the error.
10. Estimate e^3 using the first 5 terms of $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Compare to the calculator value, and find the error.

B. Estimate a definite integral:

11. Use the first 4 terms to estimate $\int_0^1 \sin(x^2)dx$ given $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
12. Use the first 4 terms to estimate $\int_0^{.5} \ln(1+x^3)\, dx$ given $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

C. Estimate a series and use an integral to find an interval that contains the exact value

13. Use the first 5 terms to estimate the sum $\sum_{n=1}^{\infty} \frac{4}{n^3}$
Use an integral find an interval that includes the exact answer.
14. Use the first 5 terms to estimate the sum $\sum_{n=1}^{\infty} \frac{6}{n^2}$
Use an integral find an interval that includes the exact answer.

III. Radius of convergence problems could be one of the following types:

A. Find the center and radius of convergence of a power series.

Sample probs. Find the center and radius of convergence for each power series:

15. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n^2 4^n}$ 16. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n4^n}$

17. $\sum_{n=0}^{\infty} \frac{x^{2n}}{9^n n!}$ 18. $\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$

B. Know what an interval of convergence means:

Sample probs:

19. A power series $f(x) = \sum_{n=0}^{\infty} a_n (x-b)^n$ has interval of convergence [2, 5). For each of these values of *x*, tell whether you can use the power series to estimate $f(x)$

a. $x = 1$ b. $x = 2$ c. $x = 3$ d. $x = 4$ e. $x = 5$ f. $x = 6$

20. A power series $f(x) = \sum_{n=0}^{\infty} a_n (x-b)^n$ has interval of convergence (-3, 1].

a. tell two values of *x* for which you can use the power series to estimate $f(x)$

b. tell two values of *x* for which the power series will not help you estimate $f(x)$.

21. If you wanted to estimate $\int_0^3 \ln(1+x)\,dx$, could you use the power series

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$

to estimate the integral? Why or why not?

IV. Show whether a series converges or diverges problems will be of one of the following types:

A. A series where a specific test is requested:

Sample probs.

22. Prove that each of these series converges or diverges using the integral test:

a. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{x^3}}$

23. Prove that this series converges or diverges using a comparison test:

a. $\sum_{n=1}^{\infty} \frac{(2n+3)}{(n^2+2n+4)}$ b. $\sum_{n=1}^{\infty} \frac{3n+1}{n^3+2n+1}$

B. A series where you should choose an appropriate test out of the series tests we have used in this chapter.

Series tests you are most likely to need are

- the ratio test,
- the comparison test and
- the alternating series test.

You should also know that if the sequence of terms being added does not converge to 0, then the series diverges.

Sample probs: show whether each converges or diverges using a test of your choice.

24. $\sum_{n=0}^{\infty} \frac{3^n}{n!}$ 25. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n^2}$

26. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ 27. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+3}$

28. $\sum_{n=1}^{\infty} \frac{(-1)^n n+1}{2n+5}$ 29. $\sum_{n=1}^{\infty} \frac{n^2+5}{n^4+3n^2+2}$

30. $\sum_{n=1}^{\infty} \frac{3n^2}{4^n}$