Math 166 review for test 3 solutions:

Do everything (max, min, increasing, decreasing, inflection points, concavity, horizontal asymptotes and graphing for:

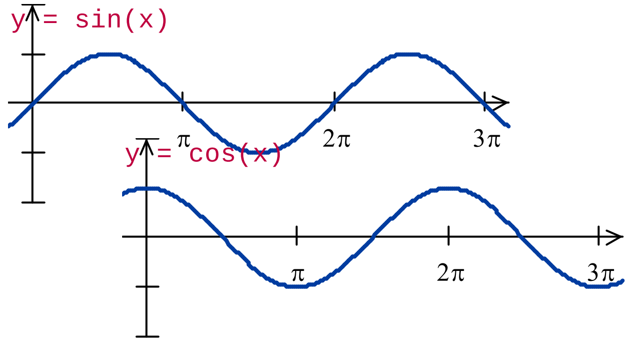
1. $y = \sin^2 x + 2\sin x$ in the interval $[0, 3\pi]$

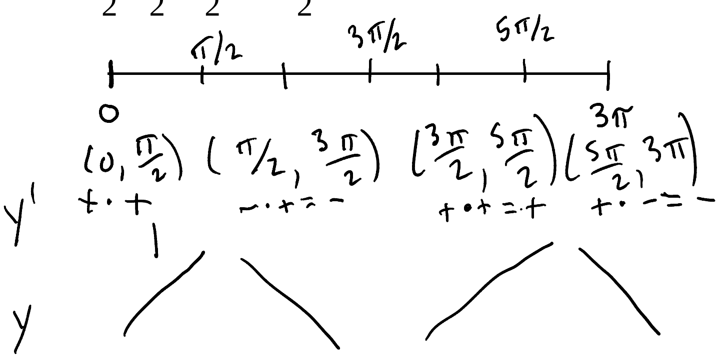
$y' = 2\sin x\cos x + 2\cos x = 0$

$2\cos x(\sin x + 1) = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \qquad \frac{3\pi}{2}$





← plug into $y' = 2\cos x(\sin x + 1)$ to get + or −

Local max: $\left(\frac{\pi}{2}, 3\right), \left(\frac{5\pi}{2}, 3\right)$ Local min: $\left(\frac{3\pi}{2}, -1\right)$

plug into $y = \sin^2 x + 2\sin x$ to get y-coordinate

$y' = 2\sin x\cos x + 2\cos x$

$y'' = 2\cos x\cos x + 2\sin x(-\sin x) + 2(-\sin x)$

$= 2\cos^2 x - 2\sin^2 x - 2\sin x$

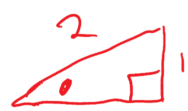
$= 2(1 - \sin^2 x) - 2\sin^2 x - 2\sin x$

$= 2 - 2\sin^2 x - 2\sin^2 x - 2\sin x$

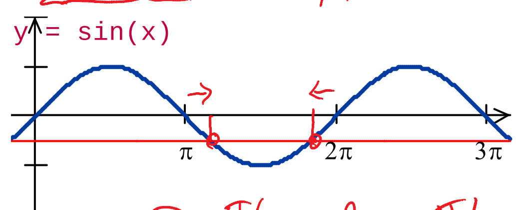
$= -4\sin^2 x - 2\sin x + 2$

$= -2(2\sin^2 x - \sin x + 1)$

$= -2(2\sin x + 1)(\sin x - 1) = 0$



$\pi/6$



$\pi + \pi/6 = \frac{7\pi}{6} \qquad 2\pi - \pi/6 = \frac{11\pi}{6}$

$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

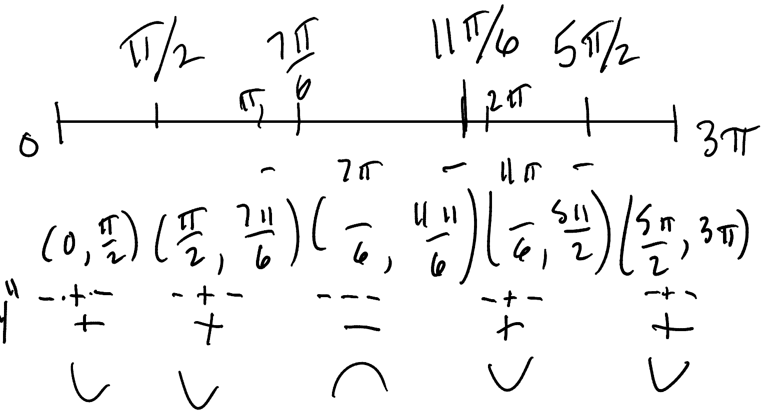
$\sin x = -\frac{1}{2} \qquad \sin x = 1$

... $\qquad x = \frac{\pi}{2}, \frac{5\pi}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$y'' = 2\cos^2 x - 2\sin^2 x - 2\sin x$$
$$= -2(2\sin x + 1)(\sin x - 1)$$

I plugged numbers into $y'' = -2(2\sin x + 1)(\sin x - 1)$ to get $\pm$ here

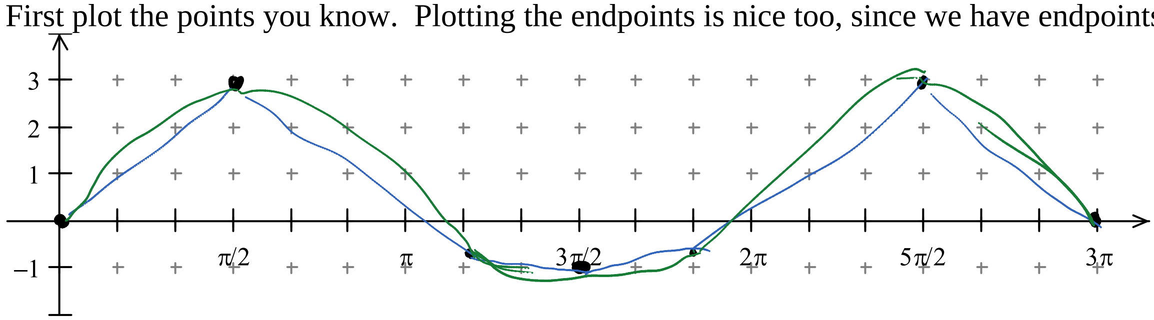


Inflection points are $\left(\frac{7\pi}{6}, -\frac{3}{4}\right), \left(\frac{11\pi}{6}, -\frac{3}{4}\right)$ ← plug into $y = \sin^2 x + 2\sin x$ to get y-coords.

No horizontal asymptote/infinite limit (the function is periodic, and the domain doesn't go all the way out for this portion).

Graph:

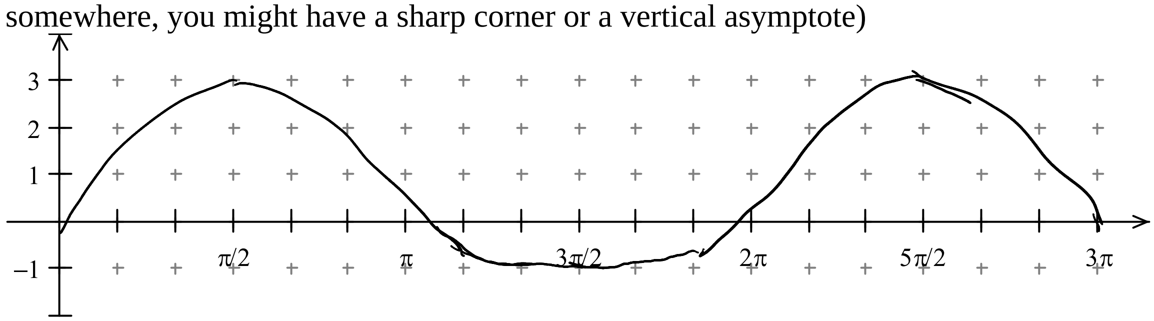
First plot the points you know. Plotting the endpoints is nice too, since we have endpoints on this one:



Local max: $\left(\frac{\pi}{2}, 3\right), \left(\frac{5\pi}{2}, 3\right)$ Local min: $\left(\frac{3\pi}{2}, -1\right)$ Inflection points $\left(\frac{7\pi}{6}, -\frac{3}{4}\right), \left(\frac{11\pi}{6}, -\frac{3}{4}\right)$

Then sketch in your info about increasing and decreasing (blue) and concavity (green)

Finally, erase that stuff that you don't need, and smooth everything out nicely (if the derivative is defined everywhere like it is for this graph, everything is smooth and curvy. If the derivative is undefined somewhere, you might have a sharp corner or a vertical asymptote)

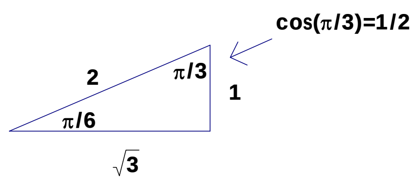


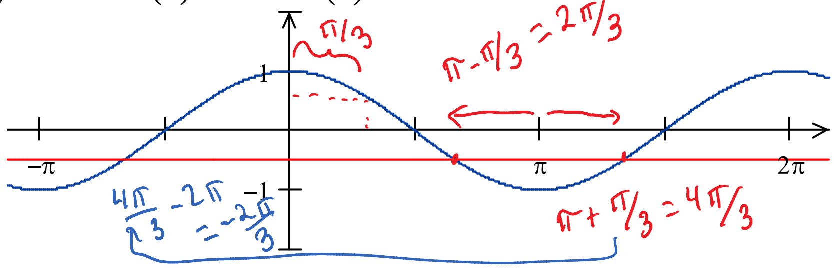
2. $\lim_{x\to-\infty}\frac{2x+5}{\sqrt{4+9x^2}}\cdot\frac{1/x}{1/x}=\lim_{x\to-\infty}\frac{2x+5}{\sqrt{4+9x^2}}\cdot\frac{1/x}{-\sqrt{1/x^2}}=\lim_{x\to-\infty}\frac{(2x+5)1/x}{\sqrt{(4+9x^2)1/x^2}}\cdot\frac{1}{-1}=\lim_{x\to-\infty}\frac{2+5/x}{-\sqrt{4/x^2+9}}=\frac{-2}{3}$

$\lim_{x\to\infty}(\sqrt{4x^2+5x}-2x)\frac{(\sqrt{4x^2+5x}+2x)}{(\sqrt{4x^2+5x}+2x)}=\lim_{x\to\infty}\frac{(4x^2+5x-4x^2)1/x}{(\sqrt{4x^2+5x}+2x)1/x}=\lim_{x\to\infty}\frac{5}{\sqrt{4+5/x}+2}=\frac{5}{4}$

3a. Find the absolute maxima and minima for: $y = x + 2\sin(x)$ $[-\pi, 2\pi]$

$y = f(x) = x + 2\sin(x)$ $[0, 2\pi]$ $y' = 1 + 2\cos(x) = 0 \Rightarrow \cos(x) = -1/2$





x	y
$-\pi$	-3.14
$-2\pi/3$	-3.83
$2\pi/3$	3.83
$4\pi/3$	2.46
2π	6.28

← Abs. min (at $-2\pi/3$)

← Abs max (at 2π)

Remember to calculate y for the end points of the interval too!

3b. Find the absolute maxima and minima for: $y = x^{7/5} - 3x^{2/5}$ $[-1, 2]$

$y = x^{7/5} - 3x^{2/5}$

$y' = \frac{7}{5}x^{2/5} - 3\cdot\frac{2}{5}x^{-3/5}$

$= \frac{x^{3/5}}{x^{3/5}}\cdot\frac{7x^{2/5}}{5} - \frac{6}{5x^{3/5}}$

$= \frac{7x-6}{5x^{3/5}}$

$7x - 6 = 0$ $\quad$ $5x^{3/5}$

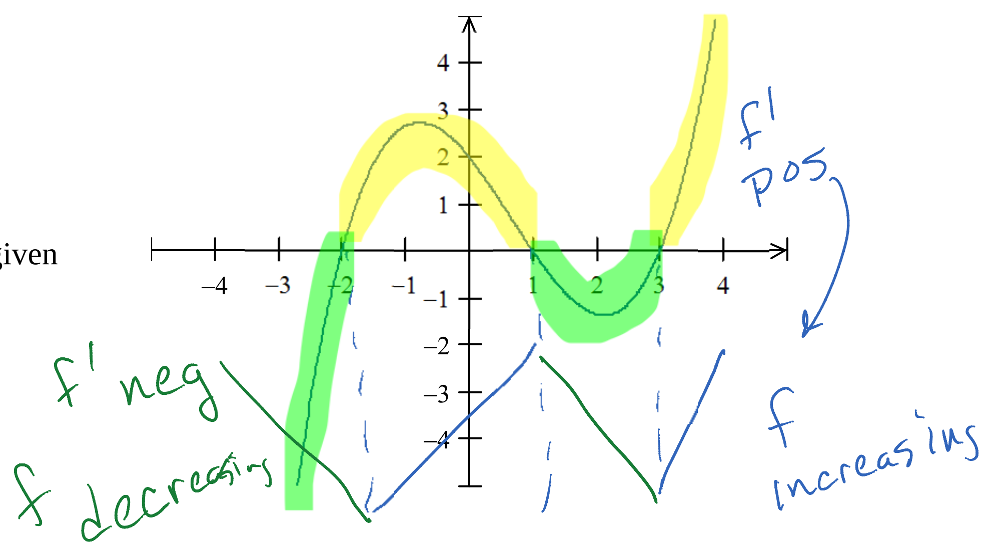
$x = 6/7$ $\quad$ $x = 0$

x	y
-1	-4
0	0
6/7	-2.01
2	-1.32

← abs. min (at -1)

← abs. max (at 0)

4. Tell the x-coordinates of the local maxima and local minima of $f(x)$, given this graph of $f'(x)$



f has a local max at $x = 1$

f has local minima at $x = -2, 3$

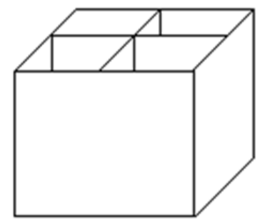
5. I want to make a box with a square base and an open top that has the greatest possible volume, with a surface area of 9ft^2. What should the dimensions of my box be?

We know A=9, so constraint is: $A = x^2 + 4xh = 9 \quad \Rightarrow h = \dfrac{9 - x^2}{4x}$.

Want max of volume: $V = x^2 h = x^2 \dfrac{9 - x^2}{4x} = \dfrac{9}{4}x - \dfrac{1}{4}x^3$

$$\Rightarrow V' = \frac{9}{4} - \frac{3}{4}x^2 = 0 \quad \Rightarrow x^2 = 3 \quad \Rightarrow x = \sqrt{3}$$

$$\Rightarrow \; h = \frac{9 - \sqrt{3}^2}{4\sqrt{3}} = \frac{9-3}{4\sqrt{3}} = \frac{6}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{4 \cdot 3} = \frac{\sqrt{3}}{2}$$



6. I want to make a box with a square base and an open top that is subdivided into 4 sections inside as shown. I need the volume of my box to be 2 ft^3. What should the dimensions of my box be so that I use the least material in constructing it? (assume that the subdivisions are made of the same material as the sides and base of the box)

V=2 ft^3 so constraint is $V = x^2 h = 2 \quad \Rightarrow h = \dfrac{2}{x^2}$.

Want min area: $A = x^2 + 6xh = x^2 + 6x\dfrac{2}{x^2} = x^2 + \dfrac{12}{x}$

$$A' = 2x - \frac{12}{x^2} = 0 \quad \Rightarrow 2x = \frac{12}{x^2} \quad \Rightarrow x^3 = 6 \quad \Rightarrow x = \sqrt[3]{6}$$

$$\Rightarrow h = \frac{2}{\left(\sqrt[3]{6}\right)^2} = \frac{2}{6^{2/3}} \cdot \frac{6^{1/3}}{6^{1/3}} = \frac{2\sqrt[3]{6}}{6} = \frac{\sqrt[3]{6}}{3}$$

7. I want to make a box with a square base and an open top that is subdivided into 4 sections inside as shown in #6. I need the volume of my box to be 2 ft^3. The sides and base of the box weigh 6oz/ft^2, and the material I use to construct the inner subdivisions weighs 3oz/ft^2. What should the dimensions of my box be so that it weighs the least?

V=2 ft^3 so constraint is $V = x^2 h = 2 \quad \Rightarrow h = \dfrac{2}{x^2}$.

Want min weight:

$$W = 6(x^2 + 4xh) + 3(2xh) = 6(x^2 + 4x \cdot \frac{2}{x^2}) + 3(2x \cdot \frac{2}{x^2}) = 6(x^2 + 4x \cdot \frac{2}{x^2}) + 3(2x \cdot \frac{2}{x^2})$$

$$= 6x^2 + 48x^{-1} + 12x^{-1} = 6x^2 + 60x^{-1}$$

$$W' = 12x - 60x^{-2} = 12x - \frac{60}{x^2} = 12x \cdot \frac{x^2}{x^2} - \frac{60}{x^2} = \frac{12x^3 - 60}{x^2}$$

$$12x^3 - 60 = 0 \Rightarrow x^3 = 5 \Rightarrow x = \sqrt[3]{5} \qquad x^2 = 0 \Rightarrow x = 0$$

$$h = \frac{2}{x^2} = \frac{2}{\sqrt[3]{5}^2} = \frac{2}{5^{2/3}} = \frac{2}{5^{2/3}} \cdot \frac{5^{1/3}}{5^{1/3}} = \frac{2\sqrt[3]{5}}{5} \approx .684$$

8. I want to make a box with a base whose length is 1.5 times its width, and with a lid whose volume is 3 ft^2. The material for the base and sides costs \$.40 per ft^2, and the cardboard for the lid costs \$.70 per ft^2. What dimensions give me the cheapest box?

V=3 ft^3 so constraint is $V = 1.5x \cdot x \cdot y = 3 \quad \Rightarrow y = \frac{3}{1.5x^2} = \frac{2}{x^2}$.

Want min cost:

$$C = .7(1.5x^2) + .4(1.5x^2 + 2 \cdot 1.5xy + 2xy) = 1.05x^2 + .6x^2 + 1.2xy + .8xy$$

$$= 1.65x^2 + 2xy = 1.65x^2 + 2x \cdot \frac{2}{x^2} = 1.65x^2 + 4x^{-1}$$

$$C' = 3.3x - 4x^{-2} = 3.3x - \frac{4}{x^2} = \frac{3.3x^3 - 4}{x^2}$$

$$3.3x^3 - 4 = 0 \Rightarrow x^3 = 4/3.3 \Rightarrow x = \sqrt[3]{4/3.3} \approx 1.06$$

$$y = \frac{2}{x^2} = \frac{2}{\sqrt[3]{4/3.3}^2} \approx 1.87$$

b. I want to make a box with a square base and an open top that is subdivided into 4 sections inside as shown in #12. I need the volume of my box to be 2 ft^3. The cardboard for the sides and base of the box costs \$.50 per ft^2, and the cardboard for the insert sections costs \$.20 per ft^2. What dimensions give me the cheapest box?

V=2 ft^3 so constraint is $V = x^2 y = 2 \quad \Rightarrow y = \frac{2}{x^2}$.

Want min cost:

$$C = .5(x^2 + 4xy) + .2(2xy) = .5x^2 + 2xy + .4xy = .5x^2 + 2.4xy = .5x^2 + 2.4x \cdot \frac{2}{x^2} = .5x^2 + 4.8x^{-1}$$

$$C' = x - 4.8x^{-2} = x - \frac{4.8}{x^2} = \frac{x^3 - 4.8}{x^2}$$

$$x^3 - 4.8 = 0 \Rightarrow x^3 = 4.8 \Rightarrow x = \sqrt[3]{4.8} \approx 1.687$$

$$y = \frac{2}{x^2} = \frac{2}{\sqrt[3]{4.8}^2} \approx .703$$