Math 166 practice exam for exam 2 solutions:

1. a. $y' = \frac{1}{2}\left(x^2+\cos^2 x\right)^{-1/2}(2x+2\cos x(-\sin x)) = \frac{2x-2\sin x\cos x}{2\sqrt{x^2+\cos^2 x}} = \frac{x-\sin x\cos x}{\sqrt{x^2+\cos^2 x}}$

b. $y' = 2\tan(3x)\sec^2(3x)(3) = 6\tan(3x)\sec^2(3x)$

c. $y = 3x^2\sin(2x) + x^3\cos(2x)\cdot 2 = 3x^2\sin(2x) + 2x^3\cos(2x)$

d. $f'(x) = 2x\sqrt{4x+3} + x^2\frac{1}{2}(4x+3)^{-1/2}\cdot 4 = 2x\sqrt{4x+3}\cdot\frac{\sqrt{4x+3}}{\sqrt{4x+3}} + \frac{2x^2}{\sqrt{4x+3}} =$
$\frac{2x(4x+3)+2x^2}{\sqrt{4x+3}} = \frac{8x^2+6x+2x^2}{\sqrt{4x+3}} = \frac{10x^2+6x}{\sqrt{4x+3}}$

e. $f'(x) = 6(3x+2)^5\cdot 3(2x-1)^8 + (3x+2)^6 8(2x-1)^7\cdot 2 = 2(3x+2)^5(2x-1)^7[9(2x-1)+(3x+2)8] =$
$2(3x+2)^5(2x-1)^7[18x-9+24x+16] = 2(3x+2)^5(2x-1)^7(42x+7)$

f. $f'(x) = \frac{3\cdot(x+2)^3 - (3x+4)3(x+2)^2\cdot 1}{(x+2)^6} = \frac{(x+2)^2[3\cdot(x+2)-(3x+4)3]}{(x+2)^6} = \frac{[3x+6-9x-12]}{(x+2)^4} = \frac{-6x-6}{(x+2)^4}$

$f(-1) = 3(-1)^8 + 3(-1)^3 + 7 = 7$

2. $f'(x) = 24x^7 + 9x^2 \qquad f'(-1) = 24(-1)^7 + 9(-1)^2 = -15$
$y = -15(x+1)+7 \qquad y = -15x-8$

3. $y = \tan x = \frac{\sin x}{\cos x} \qquad y' = \frac{\cos x\cdot\cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x+\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

4. a. $y^2 + 2xy\frac{dy}{dx} + 2\frac{dy}{dx} = 2x \quad\Rightarrow\quad (2xy+2)\frac{dy}{dx} = 2x - y^2 \quad\Rightarrow\quad \frac{dy}{dx} = \frac{2x-y^2}{2xy+2}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{2\cdot 2 - 1^2}{2\cdot 2\cdot 1 + 2} = \frac{3}{6} = \frac{1}{2} \qquad y = \frac{1}{2}(x-2)+1 \quad\Rightarrow\quad y = \frac{1}{2}x$

b.

$\sin(y) + x\cos(y)\frac{dy}{dx} = 3x^2y^2 + 2x^3y\frac{dy}{dx} \quad\Rightarrow\quad x\cos(y)\frac{dy}{dx} - 2x^3y\frac{dy}{dx} = 3x^2y^2 - \sin(y)$

$\left(x\cos(y) - 2x^3y\right)\frac{dy}{dx} = 3x^2y^2 - \sin(y) \quad\Rightarrow\quad \frac{dy}{dx} = \frac{3x^2y^2-\sin(y)}{x\cos(y)-2x^3y}$ also correct: $\frac{dy}{dx} = \frac{\sin(y)-3x^2y^2}{2x^3y - x\cos(y)}$

5. $y' = \dfrac{1\cdot(2x+3)-(x+1)\cdot 2}{(2x+3)^2} = \dfrac{2x+3-2x-2}{(2x+3)^2} = \dfrac{1}{(2x+3)^2}$

$y'' = \dfrac{0\cdot(2x+3)^2 - 1\cdot 2(2x+3)\cdot 2}{(2x+3)^4} = \dfrac{-4(2x+3)}{(2x+3)^4} = \dfrac{-4}{(2x+3)^3}$

6. $y' = -\sin(4x)\cdot 4 = -4\sin(4x)$ $y'' = -4\cos(4x)\cdot 4 = -16\cos(4x)$ $y''' = -16(-\sin(4x)\cdot 4) = 64\sin(4x)$

7. A ball tossed straight up on Mars has height: $h = -3.7t^2 + 8t + 1$ (t is measured in seconds, and h is measured in meters; give answers to at least 2 decimal points).

a. Average velocity = total distance/total time

$h(1) = -3.7 + 8 + 1 = 5.3$ $h(2) = -3.7\cdot 2^2 + 8\cdot 2 + 1 = 2.2$ $v = \dfrac{2.2-5.3}{2-1} = -3.1\,m/s$

b. $h' = -2\cdot 3.7t + 8 = -7.4t + 8$ $h'(2) = -7.4\cdot 2 + 8 = -6.8\,m/s$

c. $-7.4t + 8 = 3 \Rightarrow -7.4t = -5 \Rightarrow t \approx .676\,s.$ $h(.676) = -3.7\cdot .676^2 + 8\cdot .676 + 1 \approx 4.72$

d. $-3.7t^2 + 8t + 1 = 4 \Rightarrow -3.7t^2 + 8t - 3 = 0 \Rightarrow t = \dfrac{-8 \pm \sqrt{8^2 - 4\cdot(-3.7)\cdot(-3)}}{2\cdot(-3.7)} = .91,\ 1.25$

$h'(1.25) = -7.4\cdot 1.25 + 8 = -1.25$

e. $a = h'' = -7.4$ $h''(1.25) = -7.4$

f. $-3.7t^2 + 8t + 1 = 0 \Rightarrow t = \dfrac{-8 \pm \sqrt{8^2 - 4(-3.7)\cdot 1}}{2(-3.7)} \approx 2.28, -.12$ 2.28 seconds

8. $V = 12\cdot 14\cdot h = 168h$ $\dfrac{dV}{dh} = 168\dfrac{in^3}{in}$

9. $x_1 = 5,\ y_1 = \sin\left(\dfrac{5\pi}{6}\right) = \dfrac{1}{2}$ $y' = \dfrac{\pi}{6}\cos\left(\dfrac{\pi x}{6}\right)$ $m = \dfrac{\pi}{6}\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}\pi}{12}$

$y = -\dfrac{\sqrt{3}\pi}{12}(x-5) + \dfrac{1}{2} = -\dfrac{\sqrt{3}\pi}{12}x + \dfrac{5\sqrt{3}\pi}{12} + \dfrac{1}{2}$

10. $dP = (.15c^2 + 2)dc$ $dP = (.15\cdot 20 + 2)\cdot .5 = 2.5$ cents

11. $dy = (-4x+3)dx = (-4\cdot 1 + 3)\cdot .4 = -.4$

$y(1) = -2\cdot 1^2 + 3\cdot 1 = 1$ $y(1.4) = -2\cdot 1.4^2 + 3\cdot 1.4 = .28$ $\Delta y = .28 - 1 = -.72$