

## Math 166 practice exam for exam 2:

Expect some problems like those on the derivative quiz.

1. a.  $y = \sqrt{x^2 + \cos^2 x}$                       b.  $y = \tan^2(3x)$                       c.  $y = x^3 \sin(2x)$

Including some where you need to simplify:

d.  $f(x) = x^2 \sqrt{4x + 3}$                       e.  $f(x) = (3x + 2)^6 (2x - 1)^8$                       f.  $f(x) = \frac{3x + 4}{(x + 2)^3}$

Expect a tangent line problem

2. Find the equation of a tangent line to  $f(x) = 3x^8 + 3x^3 + 7$  at  $x = -1$

Be prepared to prove what the derivative is for  $\tan x$ ,  $\sec x$ ,  $\csc x$  or  $\cot x$ :

3. Show how to use the quotient rule to find the derivative of  $y = \tan x$

Expect 1 implicit differentiation problem:

4. a. Find the equation of the tangent line to  $xy^2 + 2y = x^2$  at the point  $(2, 1)$

b. Find  $\frac{dy}{dx}$  for  $x \sin(y) = x^3 y^2$

Expect 1-2 second or third derivative problems:

5. Find the second derivative of  $y = \frac{x+1}{2x+3}$

6. Find the third derivative of  $y = \cos 4x$

Expect 1 rate of change problems, possibly using speed or acceleration

7. A ball tossed straight up on Mars has height:  $h = -3.7t^2 + 8t + 1$  ( $t$  is measured in seconds, and  $h$  is measured in meters; give answers to at least 2 decimal points).

a. What is the average velocity between  $t=1$  and  $t=2$ ?

b. What is the instantaneous velocity at  $t=2$ ?

c. What is the height, when the ball is going up at a speed of 3m/s?

d. What is the velocity of the ball when it is 4 meters high on its way back down?

e. What is its acceleration when it is 4 meters high on its way back down?

f. How long is it in the air before it lands?

8. I have a trash compactor (with a rectangular base) that has a base of 12x14 inches. What is the rate of change of volume with respect to height as the height decreases as my garbage is compacted? Include the units of the rate of change in your answer.

Expect 1 related rates similar to those on that were assigned)

[go back and re-work the problems from the related rates assignment sheet.]

Expect a differentials problem similar to one of these:

9. (A linearization) Find the linearization of the function  $y = \sin(\pi x / 6)$  at  $a=5$ .

10. (An application) The price of a pumpkin depends on its circumference using the formula  $P = .05c^3 + 2c$ . Use a differential to approximate how much you might be undercharging for a pumpkin which you have measured to be at most 20 inches around if your measurement might be off by as much as .5 inches. ( $P$  is price in cents—sorry, this isn't a very realistic function)

11. (An explanation) Compute  $\Delta y$  and  $dy$  for the function  $y = -2x^2 + 3x$ , and the values:  $x=1$ ,  $dx = \Delta x = .4$  Sketch a diagram showing the line segments with lengths  $dx$ ,  $dy$ , and  $\Delta y$