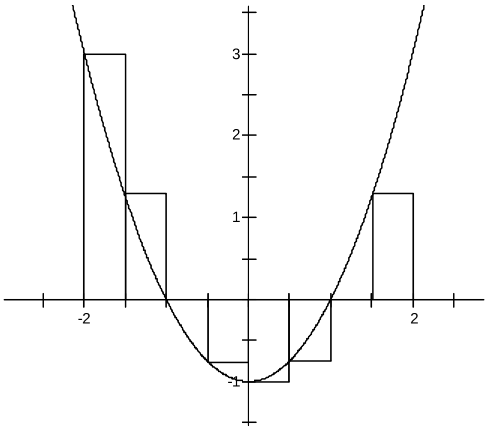
Final exam review some answers:

1.



b. $(((-2)^2-1)+((-1.5)^2-1)+((-1)^2-1)+((-.5)^2-1)+(0^2-1)+(.5^2-1)+(1^2-1)+(1.5^2-1))\cdot 0.5=1.5$

c. Several answers are correct, the most obvious being to use more rectangles.

2. a. $\int x^{1/3}(2x+1)dx=\int\ 2x^{4/3}+x^{1/3}dx=2\cdot\frac{3}{7}x^{7/3}+\frac{3}{4}x^{4/3}+C=\frac{6}{7}x^{7/3}+\frac{3}{4}x^{4/3}+C$

b. $\int\frac{3}{\sqrt{x}}-\frac{5}{x^2}+\csc x\cot x\ dx=\int\ 3x^{-1/2}-5x^{-2}dx=6x^{1/2}+\frac{5}{x}-\csc x+C$

c. $u=\csc x \qquad du=-\csc x\cot x\,dx \qquad -du=\csc x\cot x\,dx$

$x=\pi/4 \quad u=\csc(\pi/4)=\sqrt{2} \quad x=\pi/2 \quad u=\csc(\pi/2)=1$

$$\int_{\pi/4}^{\pi/2}\cot x\csc^4 x\ dx=\int_{\sqrt{2}}^{1}u^3(-1)du=-\frac{1}{4}u^4\Big|_{\sqrt{2}}^{1}=-\frac{1}{4}+\frac{4}{4}=\frac{3}{4}$$

d. $u=x^2 \quad du=2x\,dx \quad \frac{1}{2}du=x\,dx \quad x=0 \quad u=0 \quad x=0 \quad u=\pi$

$$\int_0^{\sqrt{\pi}}x\sin(x^2)\ dx=\int_0^{\pi}\sin(u)\frac{1}{2}du=-\frac{1}{2}\cos(u)\Big|_0^{\pi}=-\frac{1}{2}(-1)+\frac{1}{2}\cdot 1=1$$

e. $\int_{-\pi/2}^{\pi/2}\csc(x)\ dx$ can't be done with the fundamental theorem of calculus because csc(x) has a vertical asymptote at 0

f. $u=\sin x \quad du=\cos x\,dx$

$$\int\sqrt{\sin x}\cos x\ dx=\int\sqrt{u}\ du=\frac{2}{3}u^{3/2}+C=\frac{2}{3}\sin^{3/2}(x)+C$$

g. $u=x^2+4 \quad du=2x\,dx \quad \frac{1}{2}du=x\,dx$

$$\int\frac{x}{(x^2+4)^6}+4x\ dx=\int\frac{x}{(x^2+4)^6}dx+\int 4x\,dx=\int\frac{1}{u^6}\frac{1}{2}du+4\cdot\frac{1}{2}x^2+C$$

$$=\frac{1}{-5}\cdot\frac{1}{2}u^{-5}+2x^2+C=-\frac{1}{10(x^2+4)^5}+2x^2+C$$

h. $u=x+2 \quad du=dx \quad x=u-2$

$$\int\ \frac{x}{\sqrt{x+2}}\ dx=\int\frac{u-2}{\sqrt{u}}du=\int u^{1/2}-2u^{-1/2}\ du=\frac{2}{3}u^{3/2}-4u^{1/2}+C=\frac{2}{3}(x+2)^{3/2}-4(x+2)^{1/2}+C$$

3. (Derivative of an integral problems) Find the derivative of:

a. $g(x) = \int_{\sin(x)}^{2} \sec(2t)\,dt$ $\quad \frac{dg}{dx} = -\sec(2\sin(x))\cos(x)$ This is $\sec(2t)$ with $\sin(x)$ plugged in for t, multiplied by the derivative of $\sin(x)$ (chain rule), and multiplied by (-1), because $\sin(x)$ is the lower limit, not the upper limit.

b. $g(x) = \int_{-2}^{\sqrt{x}} \sqrt{t^4+5}\,dt$ $\quad \frac{dg}{dt} = \sqrt{x^2+5}\cdot\frac{1}{2}x^{-1/2}$ This is $\sqrt{t^4+5}$ with $\sqrt{x}$ plugged in, and multiplied by the derivative of $\sqrt{x}$

4. Using integrals/anti-derivatives to use a velocity function to find a position function:

a. If the velocity of an object is: $v(t) = \sin(\pi t/5)$ (ft/sec), and s(0)=3, find the function that tells the position of the object.

Find the anti-derivative of v:

$$\int \sin(\pi t/5)\,dt \qquad u = \pi t/5 \quad du = \pi/5\,dt \quad 5/\pi\,du = dt$$

$$\int (5/\pi)\sin(u)du = (5/\pi)(-\cos u) + C = -(5/\pi)\cos(\pi t/5) + C$$

then substitute to find C:

$$v(t) = -(5/\pi)\cos(\pi t/5) + C$$

$$v(0) = -(5/\pi)\cos(0) + C = -5/\pi + C = 3$$

$$C = 3 + 5/\pi$$

$$s(t) = -\frac{5}{\pi}\cos(\pi t/5) + 3 + \frac{5}{\pi}$$

b. Use the function you found in 4 to figure out the net distance the weight moves in the first 3 seconds (between t=0 and t=3)

$$s(3) - s(0) = -\frac{5}{\pi}\cos\left(\frac{3\pi}{5}\right) + 3 + \frac{5}{\pi} - \left(-\frac{5}{\pi}\cos(0) + 3 + \frac{5}{\pi}\right) = \frac{5}{\pi} - \frac{5}{\pi}\cos\left(\frac{3\pi}{5}\right) \approx 2.08$$

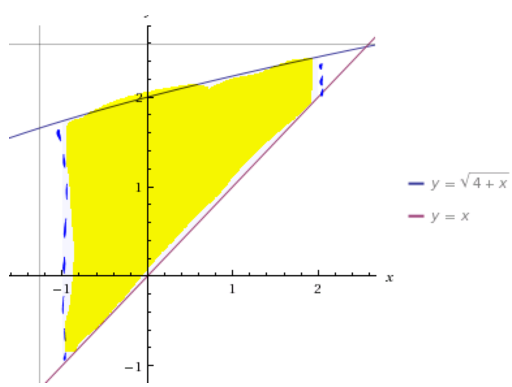
5. A problem where you must find an area by using an integral. For example:

A. Find the area bounded by $y = \sqrt{x+4}$, $y = x$, x = -1, x = 2

$$\int_{-1}^{2} \sqrt{x+4} - x\,dx = \int_{-1}^{2} \sqrt{x+4}dx - \int_{-1}^{2} x\,dx$$

$$u = x+4 \quad du = dx \quad x = -1 \Rightarrow u = 3 \quad x = 2 \Rightarrow u = 6$$

$$\int_{3}^{6} \sqrt{u}\,du - \int_{-1}^{2} x\,dx = \frac{2}{3}u^{3/2}\Big|_{3}^{6} - \frac{1}{2}x^2\Big|_{-1}^{2} \approx 4.8$$

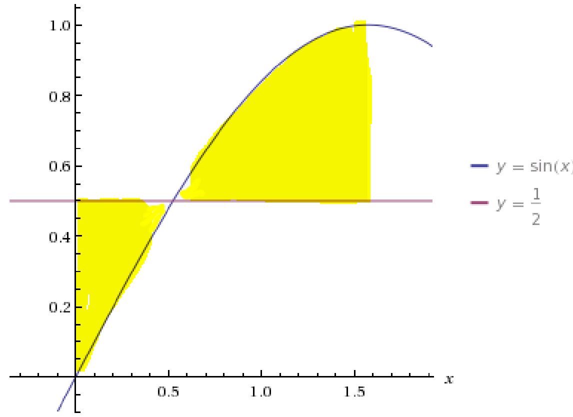


B. Find the area bounded by $y = \sin x$, $y = \frac{1}{2}$, $x = 0$, and $x = \frac{\pi}{2}$

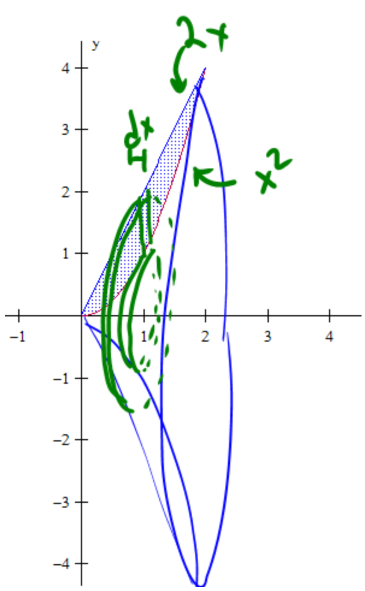
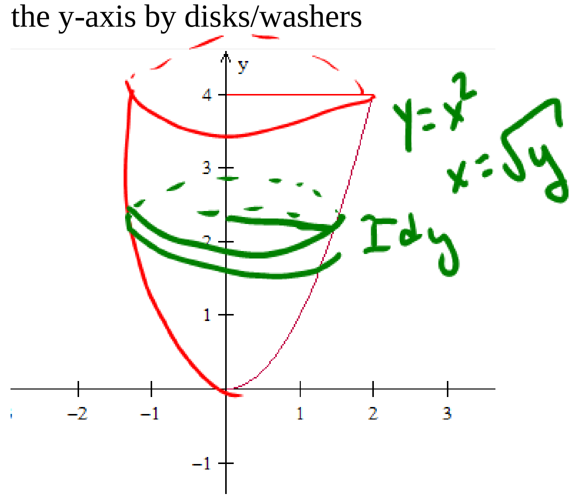
$\sin x = \frac{1}{2} \qquad x = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} \left(\frac{1}{2} - \sin(x)\right) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(-\frac{1}{2} + \sin(x)\right) dx =$$

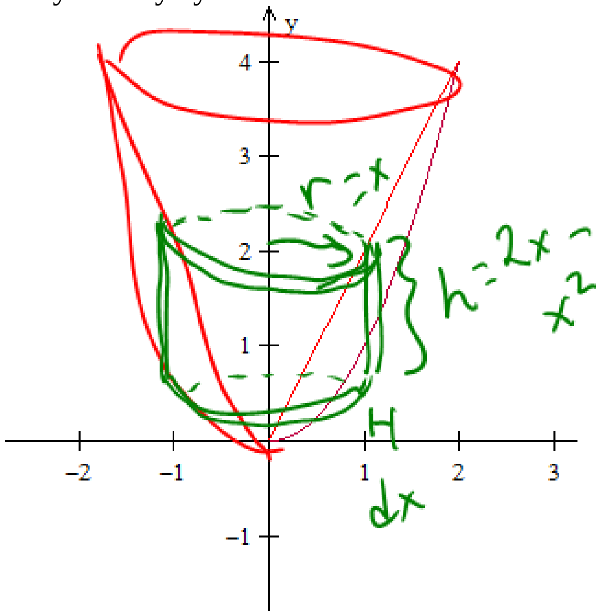
$$\frac{1}{12}\left(-12 + 12\sqrt{3} - \pi\right) \approx 0.470251$$



6. Volumes of solids by rotation. Upon due reflection, I believe I will have you set up, but not integrate a few of these (as opposed to setting up and integrating just one). For practice, set up (but don't integrate--unless you really want to)

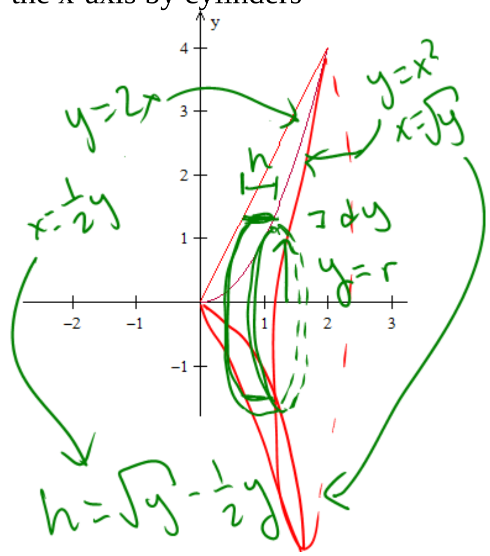
A. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 2x$ about the x-axis by disks/washers  $\int_0^2 \pi(2x)^2 - \pi(x^2)^2\, dx$	B. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 4$, $x = 0$ around the y-axis by disks/washers  $\int_0^4 \pi(\sqrt{y})^2\, dy$

C. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 2x$ about the y-axis by cylinders



$$\int_0^2 2\pi x(2x - x^2)\,dx$$

D. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 2x$ about the x-axis by cylinders



$$\int_0^4 2\pi y(\sqrt{y} - \frac{1}{2}y)dy$$

7. Work. I think I will put a rope-type work problem on the test. How about this one: rope that is 20 ft. long has a 10 lb. weight attached to the end of it, and it is hanging over a wall or building somewhere. The rope weighs 1/3 lb. per ft.. How much work is done in pulling it all up to the top of the wall

After you have pulled up y feet of rope, there remains 20-y feet left to be pulled up, and that rope weighs $\frac{1}{3}(20 - y)$, so the total weight you are pulling at that moment is $\frac{1}{3}(20 - y) + 10$ lbs. You do that for all 20 feet, so the total work is $\int_0^{20} \frac{1}{3}(20 - y) + 10 \; dy = \frac{800}{3}$

8. Averages. Find the average value of the function $y = \sin x$ on the interval $[0, \pi/3]$

$$\frac{1}{\pi/3}\int_0^{\pi/3} \sin(x)dx = \frac{3}{\pi}(-\cos(x))\Big|_0^{\pi/3} = \frac{3}{\pi}(-\frac{1}{2} + 1) = \frac{3}{2\pi}$$