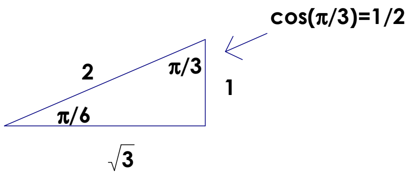
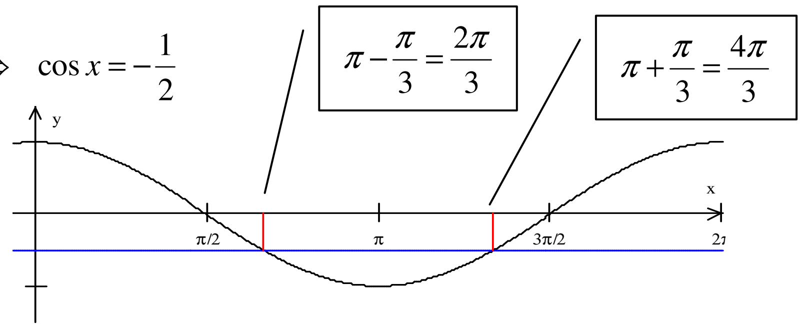
166 chapter 4 review and practice exam solutions. Practice for Friday's problems:

1. For the equation: $y = x + 2\sin(x)$ on the interval $[0, 2\pi]$

$y' = 1 + 2\cos(x)$

$1 + 2\cos x = 0 \;\Rightarrow\; 2\cos x = -1 \;\Rightarrow\; \cos x = -\frac{1}{2}$





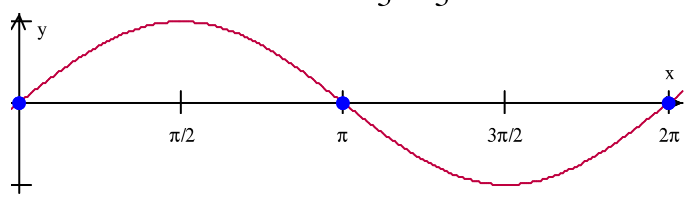
critical numbers (where the first derivative might change sign) are $\frac{2\pi}{3}, \frac{4\pi}{3}$

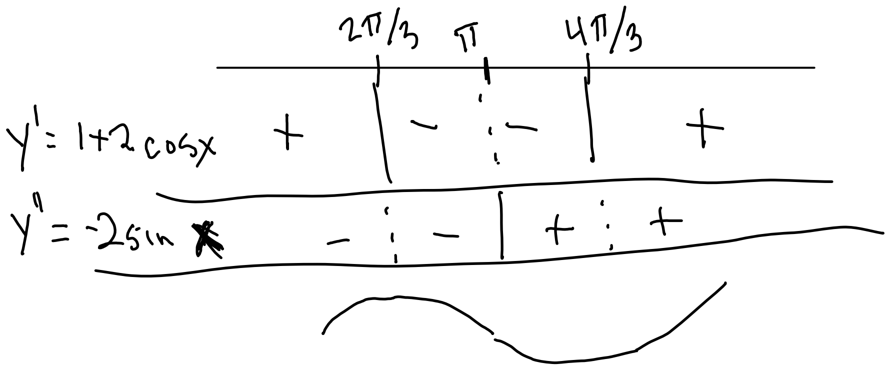
$y'' = -2\sin(x)$

$-2\sin x = 0 \;\Rightarrow\; \sin x = 0$

$x = 0,\ \pi,\ 2\pi$

numbers where y" might change sign

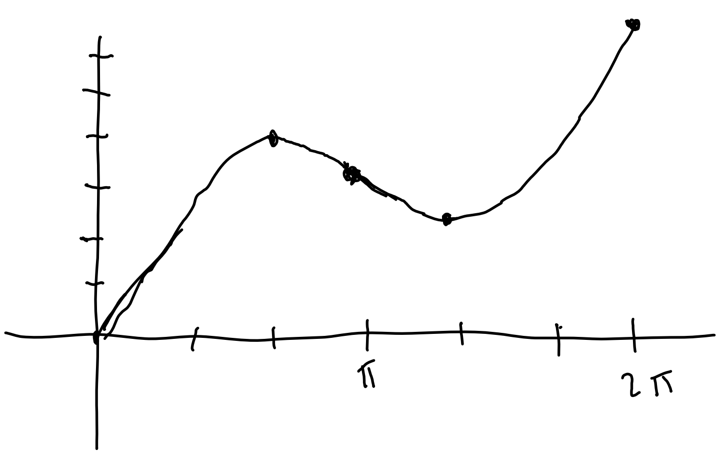




local max: $\left(\frac{2\pi}{3}, 3.83\right)$ local min: $\left(\frac{4\pi}{3}, 2.46\right)$ inflection point $(\pi, 3.14)$

endpoints (0,0), $(2\pi, 6.28)$

absolute max: $(2\pi, 6.28)$ absolute min (0,0)



2. For the equation: $f(x) = .25x^4 - 5x^2 + 3$

$f' = x^3 - 10x = x(x^2 - 10) = 0$

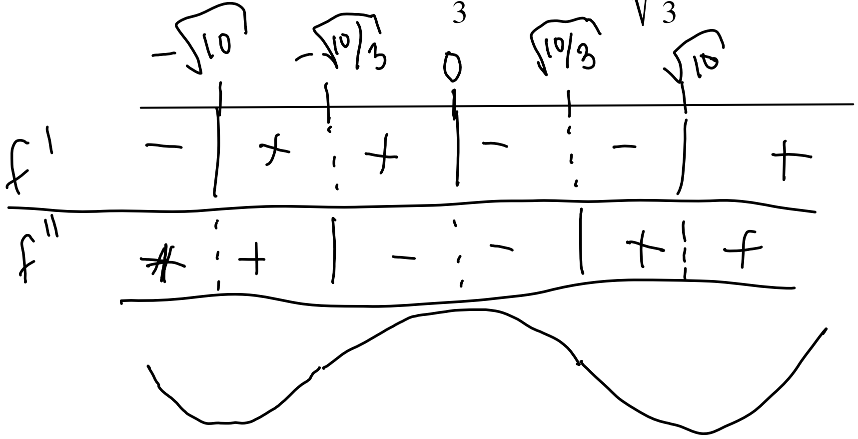
$x = 0$ critical numbers (where f' might change signs)

$x^2 - 10 = 0 \;\Rightarrow\; x^2 = 10 \;\Rightarrow\; x = \pm\sqrt{10}$

$f'(x) = x^3 - 10x$

$f''(x) = 3x^2 - 10$ values where f'' might change signs

$3x^2 - 10 = 0 \;\Rightarrow\; 3x^2 = 10 \;\Rightarrow\; x^2 = \frac{10}{3} \;\Rightarrow\; x = \pm\sqrt{\frac{10}{3}}$



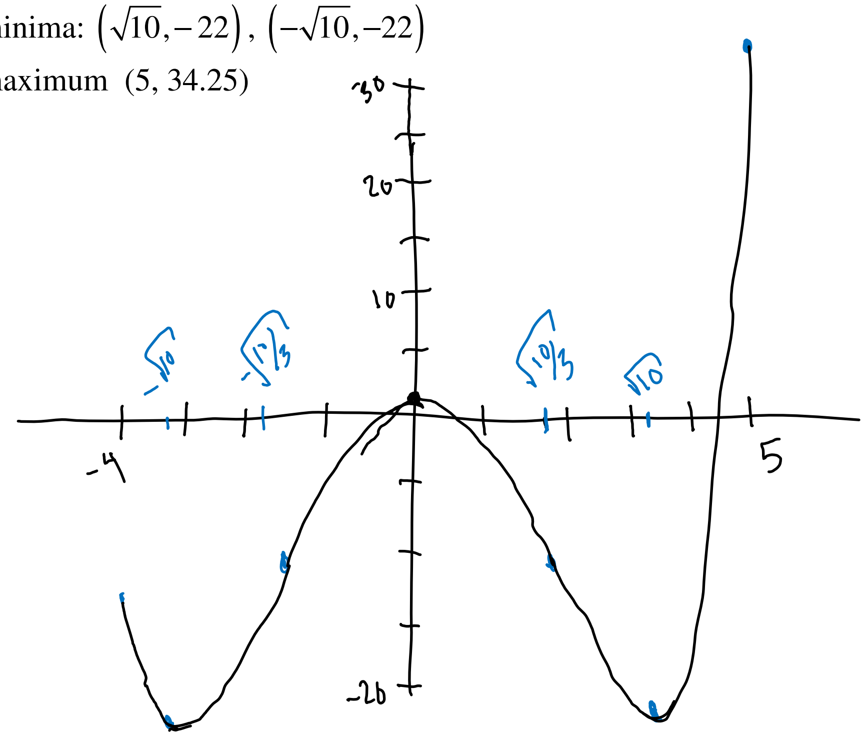
local max: (0,3), local minima: $\left(\sqrt{10}, -22\right)$, $\left(-\sqrt{10}, -22\right)$

inflection points $\left(\sqrt{\frac{10}{3}}, -10.89\right)$, $\left(-\sqrt{\frac{10}{3}}, -10.89\right)$

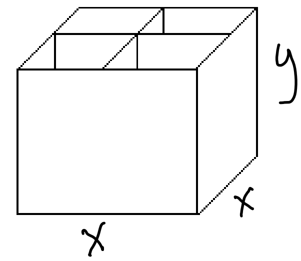
endpoints (-4, -13), (5, 34.25)

absolute minima: $\left(\sqrt{10}, -22\right)$, $\left(-\sqrt{10}, -22\right)$

absolute maximum (5, 34.25)



3. I want to make a box with a square base and an open top that is subdivided into 4 sections inside as shown. I need the volume of my box to be 2 ft^3. The cardboard for the sides and base of the box costs \$.50 per ft^2, and the cardboard for the insert sections costs \$.20 per ft^2. What dimensions give me the cheapest box?



V=2 ft^3 so constraint is $V = x^2 y = 2 \quad \Rightarrow y = \frac{2}{x^2}$.

Want min cost:

$$C = .5(x^2 + 4xy) + .2(2xy) = .5x^2 + 2xy + .4xy = .5x^2 + 2.4xy = .5x^2 + 2.4x \cdot \frac{2}{x^2} = .5x^2 + 4.8x^{-1}$$

$$C' = x - 4.8x^{-2} = x - \frac{4.8}{x^2} = \frac{x^3 - 4.8}{x^2}$$

$$x^3 - 4.8 = 0 \Rightarrow x^3 = 4.8 \Rightarrow x = \sqrt[3]{4.8} \approx 1.687$$

$$y = \frac{2}{x^2} = \frac{2}{\sqrt[3]{4.8}^2} \approx .703$$

verify that this is a local min:

$$C'' = 1 - 4.8 \cdot (-2)x^{-3} = 1 + \frac{9.6}{x^3}$$

if x=1.687, C” is positive so $\cup$ local min

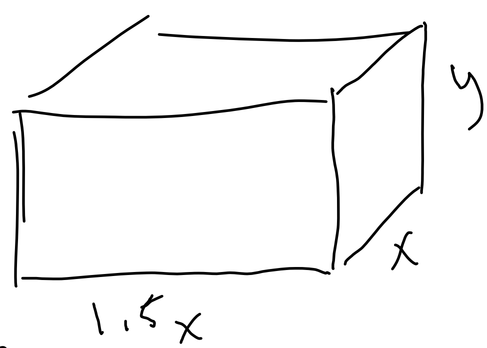
OR

1.687

C' − +

Dimensions to get the least cost are 1.69 ft. x 1.69 ft. x 0.70 ft

4. I want to make a box with a base whose length is 1.5 times its width, and with a lid whose volume is 3 ft^2. The material for the base and sides costs \$.40 per ft^2, and the cardboard for the lid costs \$.70 per ft^2. What dimensions give me the cheapest box?



V=3 ft^3 so constraint is $V = 1.5x \cdot x \cdot y = 3 \quad \Rightarrow y = \frac{3}{1.5x^2} = \frac{2}{x^2}$.

Want min cost:

$$C = .7(1.5x^2) + .4(1.5x^2 + 2 \cdot 1.5xy + 2xy) = 1.05x^2 + .6x^2 + 1.2xy + .8xy$$

$$= 1.65x^2 + 2xy = 1.65x^2 + 2x \cdot \frac{2}{x^2} = 1.65x^2 + 4x^{-1}$$

$$C' = 3.3x - 4x^{-2} = 3.3x - \frac{4}{x^2} = \frac{3.3x^3 - 4}{x^2}$$

$$3.3x^3 - 4 = 0 \Rightarrow x^3 = 4/3.3 \Rightarrow x = \sqrt[3]{4/3.3} \approx 1.06$$

1.5·1.06

$$y = \frac{2}{x^2} = \frac{2}{\sqrt[3]{4/3.3}^2} \approx 1.87$$

Verify that this is local min:

$$C'' = 3.3 - 4 \cdot (-2)x^{-3} = 3.3 + \frac{8}{x^3}$$

If x=1.06, C” is positive so $\cup$ local min

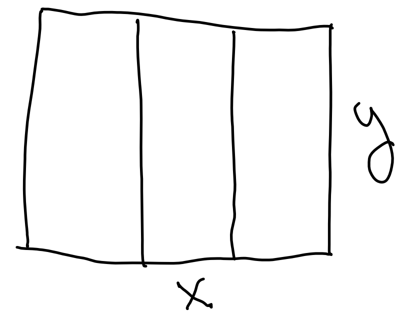
1.06

OR

$C' = 3.3x - 4x^{-2}$ − + min

Cheapest dimensions are 1.06 ft x 1.59 ft x 1.87 ft.

5. Kelly has a kennel where she raises poodles. She wants to fence in an area for the dogs to play, and she has 3 groups of dogs that she needs to keep separated. She plans to do this by fencing a rectangular area, and then subdividing it with two fences parallel to one of the sides. What is the maximum total area she can fence this way, if she has 300 ft of fencing?



She has 300 ft of fencing , so the constraint is:

$$2x+4y=300 \quad \Rightarrow \quad 2x=300-4y \quad \Rightarrow \quad x=\frac{300-4y}{2}=\frac{300}{2}-\frac{4y}{2}=150-2y$$

Want max area:

$$A=xy=(150-2y)\,y=150y-2y^2$$

$$A'=150-4y$$

$$150-4y=0 \;\Rightarrow\; 4y=150 \;\Rightarrow\; y=\frac{150}{4}=37.5$$

37.5

Verify that this is a maximum:

$A''=-4$ which is always negative, so max OR A': + −

150−4y max

The problem asks for the maximum area. When y=37.5 (plug into A), then the area is 2812.5 ft^2.