

166 chapter 4 review and practice exam. Practice for Thursday's problems:

1. For the equation: $y = x^{2/5}(x - 3)$ on the interval $[-2, 4]$

- Find the roots and vertical asymptotes if any
- Find the critical numbers (where the first derivative might change signs)
- Find the values where the second derivative might change signs
- Make a table showing where the first and second derivative are positive and negative
- Sketch the general shape of the graph
- Find y-coordinates and label the local minima, local maxima and points of inflection
- Sketch the graph of the function that includes/reflects the information you found above.
- Find and tell the absolute maximum and absolute minimum on the interval

2. For the equation $y = \frac{x}{(x+3)^2}$

- Find the roots and vertical asymptotes if any
- Find and prove the horizontal asymptote by using/finding an infinite limit
- Find the critical numbers (where the first derivative might change signs)
- Find the values where the second derivative might change signs
- Make a table showing where the first and second derivative are positive and negative
- Sketch the general shape of the graph
- Find y-coordinates and label the local minima, local maxima and points of inflection
- Sketch the graph of the function that includes/reflects the information you found above.

Other typical, likely rational functions are:

$$y = \frac{x}{x^2 + 3} \quad y = \frac{x+3}{x-2} \quad y = \frac{3}{x^2 + 4}$$

3. Find the infinite limits

- $\lim_{x \rightarrow \infty} \frac{\sqrt{4+9x^2}}{2x+5}$
- $\lim_{x \rightarrow -\infty} \frac{2x+5}{\sqrt{4+9x^2}}$
- $\lim_{x \rightarrow \infty} \sqrt{4x^2 + 5x} - 2x$

4. Sketch a graph that satisfies:

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= 3 & \lim_{x \rightarrow 2^+} f(x) &= -1 \\ f'(x) &> 0 & \text{for } x < -3, 0 < x < 2 \\ f'(x) &< 0 & \text{for } -3 < x < 0 \\ f'(x) &= 0 & \text{for } x = -3, x = 0, x > 2 \\ f''(x) &< 0 & \text{for } x < -1.5 \\ f''(x) &> 0 & \text{for } -1.5 < x < 2 \end{aligned}$$

5. Sketch a continuous graph that satisfies:

$$\begin{aligned} f(0) &= 1 \\ f'(0) &\text{ is undefined} & f'(2) &= 0 \\ f'(x) &< 0 & \text{for } 0 < x < 2 \\ f'(x) &> 0 & \text{for } x < 0, x > 2 \\ f''(x) &< 0 & \text{for } x > 3 \\ f''(x) &> 0 & \text{for } x < 0, 0 < x < 3 \\ \lim_{x \rightarrow \infty} f(x) &= -1 & \lim_{x \rightarrow -\infty} f(x) &= 0 \end{aligned}$$