To be ready for the test, you should definitely be able to calculate integrals both with and without u-substitutions. It helps to know which is which. Test your instincts.

1. Which of these problems need a u-substitution?

a. $\int e^{-x}dx$ b. $\int \frac{4}{x}dx$ c. $\int \frac{4}{x+3}dx$ d. $\int \frac{x}{3}dx$ e. $\int \frac{x+3}{2}dx$

f. $\int \sqrt{x}dx$ g. $\int \sqrt{3x}\,dx$ h. $\int \sqrt{x+3}\,dx$ i. $\int \frac{3x}{5}dx$ j. $\int e^{x+4}dx$

Check your answers on the answers page.

Here are some to practice on actually computing:

2. a. $\int_1^3 e^{-x}+\frac{3}{x}\,dx$ b. $\int_0^4 x\sqrt{2x^2+4}\,dx$ c. $\int_1^5 \frac{5}{x}+\frac{x}{5}dx$

d. $\int_0^2 (x^2-3x)-(x^3-x-4)\,dx$

You should understand how to use an integral to find an area, an average and a rolling average:

You might or might not be asked to actually find the integral depending on how long the test looks to me. If I want you to find the integral, I will say "find the area/average/rolling-average". If I just want you to set up the integral, I will tell you to "set up the integral(s) to find the area/average/rolling average, but do not evaluate". Setting up the integral means that you write out the integral that would get you the area. Evaluating means actually integrating and putting in numbers and finding a numerical answer. You should at least practice setting up the integrals that would find these areas. If you want more practice integrating, you can do that step too.

3. Area:

a. Between the curves	b. Between the curves:
$y=2x^2+3x-6$	$y=2x^2-3x-2$
$y=-x^2-3x+3$	$y=-x+2$ in the interval [1,4]

Graph and shade the region, set up the integral and (possibly) evaluate.

4. The average of $f(x)=\sqrt{2x+1}$ over the interval [0,4]
(set up and perhaps evaluate)

5. The 3-unit rolling average of $f(x)=x^3-2x^2-x+3$
(set up and perhaps evaluate)

You should also be able to interpret what an integral finds. An integral adds up or accumulates incremental values/per-unit-time amounts/rates and finds a total. If you divide a total by an appropriate amount, you may be finding an average.

6. If f(t) is the cost per month of operating a piece of equipment, and t is time in months since the beginning of 2010

a. What does $\int_0^{36} f(t)dt$ represent?

b. What does $\frac{1}{12}\int_{12}^{24} f(t)dt$ represent?

7. If f(t) is the quarterly revenue from US sales of a product, and g(t) is the quarterly revenue from European sales of a product, and t is time in years since the beginning of 2000"

a. What does $\int_4^8 f(t)-g(t)dt$ represent?

b. If $\int_4^8 f(t)-g(t)dt$ is negative, what does that mean?

8. Find y

a. $\frac{dy}{dx}=\frac{(2x+3)}{y^2}$; y = 2 when x = 0

b. $\frac{dy}{dx}=\frac{e^{3x}}{y}$

c. $\frac{dy}{dx}=2xy$

Answers:
1. Which of these problems need a u-substitution?

a. $\int e^{-x}dx$ **needs** a substitution $u = -x$

b. $\int \frac{4}{x}dx = \int 4 \cdot \frac{1}{x}dx$ no substitution needed

c. $\int \frac{4}{x+3}dx$ **needs** a substitution $u = x+3$

d. $\int \frac{x}{3}dx = \int \frac{1}{3} \cdot xdx$ no substitution needed

e. $\int \frac{x+3}{2}dx$ A trick question kinda. You can do a substitution $u = x+3$ but you don't need to because

$\int \frac{x+3}{2}dx = \int \frac{x}{2} + \frac{3}{2}dx = \int \frac{1}{2} \cdot x + \frac{3}{2}dx$

f. $\int \sqrt{x}dx = \int x^{1/2}dx$ no substitution needed

g. $\int \sqrt{3x}\, dx$ Another tricky question. You can do a substitution $u = 3x$ but you don't need to because

$\int \sqrt{3x}\, dx = \int \sqrt{3} \cdot \sqrt{x}\, dx = \int \sqrt{3}\, x^{1/2}\, dx$

h. $\int \sqrt{x+3}\, dx$ **Needs** a substitution $u = x+3$ because $\sqrt{x+3} \neq \sqrt{x} + \sqrt{3}$

i. $\int \frac{3x}{5}dx = \int \frac{3}{5}x\, dx$ No substitution needed

j. $\int e^{x+4}dx$ Very tricky. You can do a substitution $u = x+4$ but if you know your algebra you don't have to because $e^{x+4} = e^x \cdot e^4$ and e^4 is a constant!

Not all steps are shown, but a few key steps are shown:
2. a.

$$\int_1^3 e^{-x} + \frac{3}{x}\, dx = \int_1^3 e^{-x}dx + \int_1^3 \frac{3}{x}dx$$

$\int e^{-x}dx$

$= \int e^u(-1)du$

$= -e^u + C = -e^{-x} + C$

$u = -x$

$\frac{du}{dx} = -1$

$-du = dx$

$\int_1^3 e^{-x}dx = -e^{-x}\Big|_1^3$

$-e^{-3} - (-e^{-1}) \approx .3181$

$\int_1^3 \frac{3}{x}dx$

$= \int_1^3 3 \cdot \frac{1}{x}dx$

$= 3\ln|x|\Big|_1^3$

$= 3\ln 3 - 3\ln 1 \approx 3.2958$

$$\int_1^3 e^{-x} + \frac{3}{x}\, dx \approx .3181 + 3.2958 = 3.6139$$

b. $\int_0^4 x\sqrt{2x^2+4}\,dx$

$u = 2x^2+4$

$\frac{du}{dx} = 4x$

$\frac{1}{4x}du = dx$

$\int x\sqrt{2x^2+4}\,dx$

$= \int x\sqrt{u}\,\frac{1}{4x}du$

$= \int \frac{1}{4}u^{1/2}du$

$= \frac{1}{4}\cdot\frac{2}{3}u^{3/2}+C = \frac{1}{3}\left(2x^2+4\right)^{3/2}+C$

$\int_0^4 x\sqrt{2x^2+4}\,dx = \frac{1}{3}\left(2x^2+4\right)^{3/2}\Big|_0^4 = \frac{1}{3}\left(36^{3/2}-4^{3/2}\right)$

$= \frac{208}{3} = 69\frac{1}{3} \approx 69.3333$

c. $\int_1^5 \frac{5}{x}+\frac{x}{5}\,dx = \int_1^5 5\cdot\frac{1}{x}+\frac{1}{5}\cdot x\,dx$

$= 5\cdot\ln|x| + \frac{1}{5}\cdot\frac{x^2}{2}\Big|_1^5 = 5\ln 5+\frac{25}{10}-\left(5\ln 1+\frac{1}{10}\right)$

$= 5\ln 5 + 2.4 \approx 10.4472$

d. $\int_0^2 (x^2-3x)-(x^3-x-4)\,dx$

$= \int_0^2 -x^3+x^2-2x+4\,dx$

$= \frac{-x^4}{4}+\frac{x^3}{3}-x^2+4x\Big|_0^2$

$= -4+\frac{8}{3}-4+8-(0) = \frac{8}{3}$

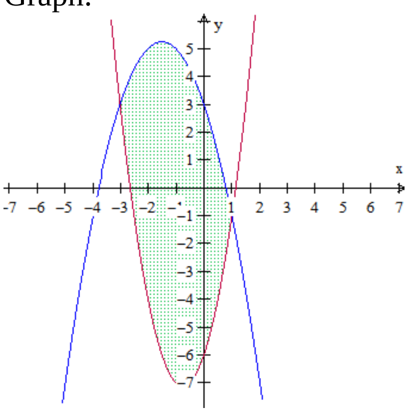
3. Area:

a. Between the curves

$y = 2x^2+3x-6$

$y = -x^2-3x+3$

Graph:



Set up:

$2x^2+3x-6 = -x^2-3x+3$

$3x^2+6x-9 = 0$

$3(x+3)(x-1) = 0$

$x = 1, -3$

$\int_{-3}^{1} -x^2-3x+3-(2x^2+3x-6)dx$

Evaluate:

$= \int_{-3}^{1} -3x^2-6x+9dx$

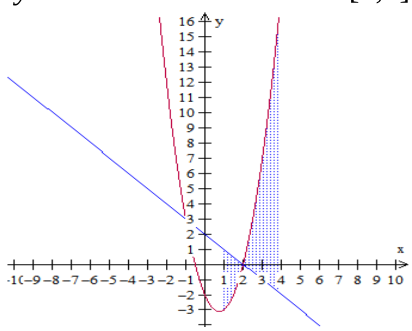
$= -x^3-3x^2+9x\Big|_{-3}^{1}$

$= -1-3+9-(27-27-27) = 32$

b. Between the curves:

$y = 2x^2-3x-2$

$y = -x+2$ in the interval [1,4]



Set up:

$2x^2-3x-2 = -x+2$

$2x^2-2x-4 = 0$

$2(x-2)(x+1) = 0$

$x = -1, 2$

$\int_1^2 -x+2-\left(2x^2-3x-2\right)dx + \int_2^4 2x^2-3x-2-\left(-x+2\right)dx$

$\int_1^2 -2x^2+2x+4\,dx$

$= \frac{-2x^3}{3}+x^2+4x\Big|_1^2$

$= \frac{-16}{3}+4+8-\left(\frac{-2}{3}+1+4\right)$

$= \frac{-14}{3}+7 = \frac{7}{3} = 3\frac{1}{3} \approx 3.3333$

$\int_2^4 2x^2-2x-4\,dx$

$= \frac{2x^3}{3}-x^2-4x\Big|_2^4$

$= \frac{128}{3}-16-16-\left(\frac{16}{3}-4-8\right)$

$= \frac{112}{3}-20 = \frac{52}{3} = 17\frac{1}{3} \approx 17.3333$

total area $= 20\frac{2}{3} \approx 20.6667$

4. The average of $f(x)=\sqrt{2x+1}$ over the interval [0,4]
(set up and perhaps evaluate)

Set up: $\frac{1}{4}\int_0^4 \sqrt{2x+1}\,dx$

Evaluate:

$u=2x+1$

$dx=\frac{1}{2}du$

$\int \sqrt{u}\,\frac{1}{2}du=\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C=\frac{1}{3}(2x+1)^{3/2}+C$

$$\frac{1}{4}\int_0^4 \sqrt{2x+1}\,dx=\frac{1}{4}\cdot\frac{1}{3}(2x+1)^{3/2}\Big|_0^4=\frac{1}{12}\left(9^{3/2}-1^{3/2}\right)=\frac{26}{12}=\frac{13}{3}=4\frac{1}{3}$$

5. The 3-unit rolling average of $f(x)=x^3-2x^2-x+3$
(set up and perhaps evaluate)

Set up:

$\frac{1}{3}\int_{x-3}^{x} t^3-2t^2-t+3\,dt$

Evaluate:

$$=\frac{1}{3}\left(\frac{t^4}{4}-\frac{2t^3}{3}-\frac{t^2}{2}+3t\right)\Bigg|_{x-3}^{x}=\frac{1}{3}\left(\frac{x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}+3x\right)-\frac{1}{3}\frac{(x-3)^4}{4}-\frac{2(x-3)^3}{3}-\frac{(x-3)^2}{2}+3(x-3)$$

The most important terms (the ones that are most likely to have points attached to them) are in bold.

6. If f(t) is the cost per month of operating a piece of equipment, and t is time in months since the beginning of 2010

a. What does $\int_0^{36} f(t)dt$ represent?

The **total cost** of operating the equipment between the beginning of **2010** and the beginning of **2013**

b. What does $\frac{1}{12}\int_{12}^{24} f(t)dt$ represent?

The **average monthly cost** of operating the equipment during **2011**.

7. If f(t) is the quarterly revenue from US sales of a product, and g(t) is the quarterly revenue from European sales of a product, and t is time in years since the beginning of 2000"

a. What does $\int_4^8 f(t)-g(t)dt$ represent?

How much more revenue came from **US Sales than European sales** of the product during **2011**

b. If $\int_4^8 f(t)-g(t)dt$ is negative, what does that mean?

It means that European sales were more than US sales.

8. Find y

a. $\frac{dy}{dx}=\frac{(2x+3)}{y^2}$; y = 2 when x = 0

$$\int y^2 dy=\int 2x+3\,dx$$

$$\frac{y^3}{3}=x^2+3x+C$$

$$y^3=3x^2+9x+C$$

$$y=\sqrt[3]{3x^2+9x+C}$$

$$2=\sqrt[3]{0+C}$$

$$8=C$$

$$y=\sqrt[3]{3x^2+9x+8}$$

b. $\frac{dy}{dx}=\frac{e^{3x}}{y}$

$$\int y\,dy=\int e^{3x}dx$$

$$u=3x$$

$$\frac{1}{3}du=dx$$

$$\int y\,dy=\int e^u\frac{1}{3}du$$

$$\frac{y^2}{2}=\frac{1}{3}e^u+C$$

$$\frac{y^2}{2}=\frac{1}{3}e^{3x}+C$$

$$y^2=\frac{2}{3}e^{3x}+C$$

$$y=\sqrt{\frac{2}{3}e^{3x}+C}$$

c. $\frac{dy}{dx}=2xy$

$$\int\frac{1}{y}dy=\int 2xdx$$

$$\ln|y|=x^2+C$$

$$y=e^{x^2+C}$$

$$y=e^{x^2}e^C$$

$$y=Ae^{x^2}$$