Review problems applying calculus to cost, revenue and profit functions

1. The annual profit of a company was

$$P(t) = t^2 + 2t + 50 \qquad (0 \le t \le 5)$$

million dollars, where *t* is number of years since 2000.

a. Find the rate of change of profit at t=3, and interpret your answer.
b. Find the total profit of the company between 2000 and 2004.
c. Find the average yearly profit of the company between 2000 and 2004.

2. The cost in dollars of producing *x* items per day of a certain product is

$$C(x) = .0025x^2 + 80x + 10{,}000 \qquad (100 \le x \le 4000\ )$$

a. Find the marginal cost when *x*=400, and interpret your answer.
b. Find the average cost when *x*=400, and interpret your answer.
b. Find the minimum average daily cost under this model. What is the average cost? What production level would give that cost?

3. The profit in dollars from daily production of *x* items is

$$P(x) = -.000002x^3 + 6x - 400$$

a. Find the marginal profit when x=400, and interpret your answer
b. Find the maximum possible daily profit under this model. What is the profit? What production level would give that profit?

4. Advertisement spending in millions of dollars for a particular company between 2005 (t=1) and 2011 (t=7) was

$$S(t) = .86t^{.96} \qquad (1 \le t \le 7\ )$$

a. Compute $\int_1^7 S(t)dt$ and interpret your answer.
b. Compute $\frac{1}{6}\int_1^7 S(t)dt$ and interpret your answer.

Comments in italics are me explaining something, and are not part of the required answer.

1. The annual profit of a company was

$$P(t) = t^2 + 2t + 50 \qquad (0 \le t \le 5)$$

million dollars, where *t* is number of years since 2000.

a. Find the rate of change of profit at t=3, and interpret your answer.

Rate of change means derivative (especially derivative with respect to time), so compute:

$$P(t) = t^2 + 2t + 50$$

$$P'(t) = 2t + 2$$

$$P'(3) = 2 \cdot 3 + 2 = 8$$

the interpretation of what 8 means in this context is: the company's profit in 2003 was increasing at a rate of 8 million dollars per year.

b. Find the total profit of the company between 2000 and 2004.

$$\int_0^4 t^2 + 2t + 50\, dt = \frac{t^3}{3} + t^2 + 50t \Big|_0^4 = \frac{4^3}{3} + 4^2 + 50 \cdot 4 - 0 = 237.33$$

The total profit was 237.33 million dollars*

c. Find the average yearly profit of the company between 2000 and 2004.

$$\frac{1}{4}\int_0^4 t^2 + 2t + 50\, dt = \frac{1}{4}\left(\frac{t^3}{3} + t^2 + 50t\right)\Big|_0^4 = \frac{1}{4}\left(\frac{4^3}{3} + 4^2 + 50 \cdot 4 - 0\right) = 59.33$$

The average yearly profit was 59.33 million dollars.*

*including units (like million dollars) is likely to be worth some small amount of points on any application problem that includes unit information in the text of the problem.

2. The cost in dollars of producing *x* items per day of a certain product is

$$C(x) = .0025x^2 + 80x + 10{,}000 \qquad (100 \le x \le 4000\ )$$

a. Find the marginal cost when *x*=400, and interpret your answer.

Marginal cost is rate of change with respect to number produced (when the variable is number of items produced, the derivative is marginal cost), so start by taking the derivative:

$$C'(x) = .005x + 80$$

$$C'(400) = .005 * 400 + 80 = 82$$

The interpretation says what this number tells about cost: It costs \$82 to produce the 401st item (after all costs have been covered for the first 400)

b. Find the average cost when *x*=400, and interpret your answer.

Average cost means to divide cost by the number produced:

$$\overline{C}(x) = \frac{.0025x^2 + 80x + 10{,}000}{x}$$

$$\overline{C}(400) = \frac{.0025 \cdot 400^2 + 80 \cdot 400 + 10{,}000}{400} = 106$$

Interpretation: when the total costs for making 400 items are spread out over all of the items, the cost is \$106 per item.

b. Find the minimum average daily cost under this model. What is the average cost? What production level would give that cost?

First simplify the average cost algebraically, and then take the derivative, set equal to zero and solve.

$$\bar{C}(x) = \frac{.0025x^2 + 80x + 10,000}{x} = \frac{.0025x^2}{x} + \frac{80x}{x} + \frac{10000}{x} = .0025x + 80 + 10000x^{-1}$$

$$\bar{C}'(x) = .0025 - 10000x^{-2} = 0$$

$$.0025 = 10000x^{-2}$$

$$.0025 = \frac{10000}{x^2}$$

$$.0025x^2 = 10000$$

$$x^2 = \frac{10000}{.0025} = 4000000$$

$$x = \sqrt{4000000} = 2000$$

Now check the average cost for this value of x and for the endpoints: 100 and 4000

x	100	2000	4000
$\bar{C}(x)$	$\frac{.0025 \cdot 100^2 + 80 \cdot 100 + 10,000}{100}$ =180.25	$\frac{.0025 \cdot 2000^2 + 80 \cdot 2000 + 10,000}{2000}$ =90	$\frac{.0025 \cdot 4000^2 + 80 \cdot 4000 + 10,000}{4000}$ =92.5

The minimum average cost is $90 per item. This average cost is attained when 90 items are produced daily.

3. The profit in dollars from daily production of *x* items is

$$P(x) = -.000002x^3 + 6x - 400$$

a. Find the marginal profit when x=400, and interpret your answer

$$P(x) = -.000002x^3 + 6x - 400$$

$$P'(x) = -.000006x^2 + 6$$

$$P'(400) = -.000006 \cdot 400^2 + 6 = 5.04$$

An profit of $5.04 is earned from producing the 401st item.

b. Find the maximum possible daily profit under this model. What is the profit? What production level would give that profit?

$P(x) = -.000002x^3 + 6x - 400$

$P'(x) = -.000006x^2 + 6 = 0$

$-.000006x^2 = -6$

$.000006x^2 = 6$

$x^2 = \dfrac{6}{.000006} = 1000000$

$x = 1000$

Test that x=1000 produces a maximum by either the first or second derivative tests:

$P'(x) = -.000006x^2 + 6$ 1,000 P' + − max	$P''(x) = -.000012x$ $P''(1000) = -.000012 \cdot 1000 = -.012 < 0$ ∩ max

When x=1000, the company will make the maximum profit. That maximum daily profit is $3600

4. Advertisement spending in millions of dollars for a particular company between 2005 (t=1) and 2011 (t=7) was

$$S(t) = .86t^{.96} \quad (1 \le t \le 7)$$

a. Compute $\int_1^7 S(t)dt$ and interpret your answer.

$$\int_1^7 .86t^{0.96}dt = .86 \cdot \frac{t^{1.96}}{1.96}\Bigg|_1^7 = .86 \cdot \frac{7^{1.96}}{1.96} - .86 \cdot \frac{1^{1.96}}{1.96} \approx 19.45$$

Interpretation: The company spent 19.45 million dollars total between 2005 and 2011

b. Compute $\frac{1}{6}\int_1^7 S(t)dt$ and interpret your answer.

Notice that 7-1=6 and 2011-2005=6, so this is an average

$$\frac{1}{6} \cdot \int_1^7 .86t^{0.96}dt = \frac{1}{6} \cdot .86 \cdot \frac{t^{1.96}}{1.96}\Bigg|_1^7 = \frac{1}{6} \cdot .86 \cdot \frac{7^{1.96}}{1.96} - \frac{1}{6} \cdot .86 \cdot \frac{1^{1.96}}{1.96} \approx 3.24$$

The company spent an average of 3.24 millions of dollars per year between 2005 and 2011.