Review:

1. Practice some derivatives with the product and chain rules. Find the derivative of each function:

a. $f(x) = x^4 \ln(x^2 + 3x)$

b. $g(x) = x^5 e^{3x+2}$

c. $h(x) = 2x\sqrt{3x+5}$

d. $y = 4\sqrt{x^2 + 3x}$

e. $y = \ln(3x+1)(x^2 - 2x)^5$

2. More partial derivatives: find the partial derivative with respect to x and the partial derivative with respect to y of:

$f(x) = y^3 \ln(x^2 + 5xy + 1)$

3. Practice finding maxima and minima with 1-variable functions. Find all of the maxima and minima of each function. If an interval is given, remember to check the endpoints and test them against other local maxima and minima to see if the endpoint is an absolute maximum or minimum or not.

a. $y = 2x^3 - 3x^2 - 12x + 1$

b. $y = 2x^2 - 2x + 3$ on [0,3]

Answers on the next page.

To be ready for the final exam, you need to be able to do problems like these without looking at your notes.

Answers:

1. Practice some derivatives with the product and chain rules. Find the derivative of each function:

a. $f(x) = x^4 \ln(x^2+3x)$

$$f'(x) = 4x^3 \ln(x^2+3x) + x^4 \cdot \frac{1}{x^2+3x} \cdot (2x+3)$$
$$= 4x^3 \ln(x^2+3x) + \frac{x^4(2x+3)}{x^2+3x}$$
$$= 4x^3 \ln(x^2+3x) + \frac{x^3(2x+3)}{x+3}$$

b. $g(x) = x^5 e^{3x+2}$

$$g'(x) = 5x^4 e^{3x+2} + x^5 e^{3x+2} \cdot 3$$
$$= 5x^4 e^{3x+2} + 3x^5 e^{3x+2}$$
$$= x^4 e^{3x+2}(5+3x)$$

c. $h(x) = 2x\sqrt{3x+5}$

$$h'(x) = 2\sqrt{3x+5} + 2x \cdot \frac{1}{2}(3x+5)^{-1/2} \cdot 3$$
$$= 2\sqrt{3x+5} + 3x(3x+5)^{-1/2}$$
$$= 2\sqrt{3x+5} + \frac{3x}{(3x+5)^{1/2}}$$
$$= 2\sqrt{3x+5} + \frac{3x}{\sqrt{3x+5}}$$

d. $y = 4\sqrt{x^2+3x}$

$$y' = 4 \cdot \frac{1}{2}(x^2+3x)^{-1/2}(2x+3)$$
$$= 2(x^2+3x)^{-1/2}(2x+3)$$
$$= \frac{2(2x+3)}{(x^2+3x)^{1/2}}$$
$$= \frac{4x+6}{\sqrt{x^2+3x}}$$

e. $y = \ln(3x+1)(x^2-2x)^5$

$$y' = \frac{1}{3x+1} \cdot 3 \cdot (x^2-2x)^5 + \ln(3x+1) \cdot 5 \cdot (x^2-2x)^4(2x-2)$$
$$= \frac{3(x^2-2x)^5}{3x+1} + 5\ln(3x+1)(x^2-2x)^4(2x-2)$$

2. More partial derivatives: find the partial derivative with respect to x and the partial derivative with respect to y of:

$$f(x) = y^3 \ln(x^2+5xy+1)$$

$$\frac{\partial f}{\partial x} = y^3 \frac{1}{x^2+5xy+1}(2x+5y) = \frac{y^3(2x+5y)}{x^2+5xy+1}$$

$$\frac{\partial f}{\partial y} = 3y^2 \ln(x^2+5xy+1) + y^3 \frac{1}{x^2+5xy+1} \cdot 5x = 3y^2 \ln(x^2+5xy+1) + \frac{5xy^3}{x^2+5xy+1}$$

3. Practice finding maxima and minima with 1-variable functions. Find all of the maxima and minima of each function. If an interval is given, remember to check the endpoints and test them against other local maxima and minima to see if the endpoint is an absolute maximum or minimum or not.

a. $y = 2x^3 - 3x^2 - 12x + 1$

Find possible max/mins	Check for max/min opt 1: first deriv:	Option 2: second deriv:
$y' = 6x^2 - 6x - 12 = 0$ $6(x^2 - x - 2) = 0$ $6(x-2)(x+1) = 0$ $x = 2, -1$	-1 2 y' $+$ $-$ $+$ max min	$y'' = 12x - 6$ x | y'' -1 | -18 ∩ max 2 | 18 ∪ min.
Find y-coords and write answer:		
x | -1 2 y | 8 -19	max $(-1, 8)$	min $(2, -19)$

b. $y = 2x^2 - 2x + 3$ on [0,3]

<table>
<tr>
<td>Find possible max/mins
$y' = 4x - 2 = 0$
$x = \frac{1}{2}$</td>
<td>Check for max/min opt 1: first deriv:
$\frac{1}{2}$
y' −   +
\ min /</td>
<td>Option 2: second deriv:
$y'' = 4$
U min</td>
</tr>
<tr>
<td colspan="3">Find y-coords including endpoints and write answer:
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>.5</td><td>2.5</td></tr>
<tr><td>3</td><td>15</td></tr>
</table>
min (local and absolute) $(\frac{1}{2}, 2\frac{1}{2})$
absolute max (3,15)</td>
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