

Practice with exponential functions with bases other than e:

1. Find the derivatives:

a. $f(x) = 8^x$

b. $g(x) = 17^x$

2. Find the integrals:

a. $\int 2^x dx$

b. $\int_{-1}^1 4^x dx$

c. $\int_{-1}^2 5^x dx$

Answers on next page.

1. Find the derivatives:

a. $f(x) = 8^x = e^{\ln(8) \cdot x}$

$$f'(x) = e^{\ln(8) \cdot x} \cdot \ln(8) = 8^x \cdot \ln(8)$$

b. $g(x) = 17^x = e^{\ln(17) \cdot x}$

$$g'(x) = e^{\ln(17) \cdot x} \cdot \ln(17) = 17^x \cdot \ln(17)$$

2. Find the integrals:

a. $\int 2^x dx = \int e^{\ln(2) \cdot x} dx$

$$= \int e^u \cdot \frac{1}{\ln(2)} du$$

$$= e^u \cdot \frac{1}{\ln(2)} + C = \frac{1}{\ln(2)} e^{\ln(2) \cdot x} + C$$

$$= \frac{2^x}{\ln(2)} + C$$

$$u = \ln(2) \cdot x$$

$$\frac{du}{dx} = \ln(2)$$

$$du = \ln(2) \cdot dx$$

$$\frac{1}{\ln(2)} du = dx$$

b. $\int_{-1}^1 4^x dx = \int_{-1}^1 e^{\ln(4) \cdot x} dx$

$$\int e^u \cdot \frac{1}{\ln(4)} du = \frac{e^u}{\ln(4)} + C$$

$$= \frac{e^{\ln(4) \cdot x}}{\ln(4)} + C = \frac{4^x}{\ln(4)} + C$$

$$\int_{-1}^1 4^x dx = \left. \frac{4^x}{\ln(4)} \right|_{-1}^1 = \frac{4}{\ln(4)} - \frac{4^{-1}}{\ln 4} \approx 2.885 - .180 = 2.705$$

$$u = \ln(4) \cdot x$$

$$\frac{du}{dx} = \ln(4)$$

$$du = \ln(4) dx$$

$$\frac{1}{\ln(4)} du = dx$$

c. $\int_{-1}^2 5^x dx = \int_{-1}^2 e^{\ln(5) \cdot x} dx$

$$\int e^u \cdot \frac{1}{\ln(5)} du = \frac{e^u}{\ln(5)} + C$$

$$= \frac{e^{\ln(5) \cdot x}}{\ln(5)} + C = \frac{5^x}{\ln(5)} + C$$

$$\int_{-1}^2 5^x dx = \left. \frac{5^x}{\ln(5)} \right|_{-1}^2 = \frac{5^2}{\ln(5)} - \frac{5^{-1}}{\ln(5)} = \frac{25}{\ln(5)} - \frac{.2}{\ln(5)} \approx 39.914$$

$$u = \ln(5) \cdot x$$

$$\frac{du}{dx} = \ln(5)$$

$$du = \ln(5) dx$$

$$\frac{1}{\ln(5)} du = dx$$