1. For the function $f(x,y) = x^3 + 3y^2 - 4xy^2 + 4x$

a. Find $\frac{\partial f}{\partial x}$ b. Find $\frac{\partial f}{\partial y}$ c. Find $\frac{\partial^2 f}{\partial x^2}$ d. Find $\frac{\partial^2 f}{\partial y^2}$ e. Find $\frac{\partial^2 f}{\partial x \partial y}$

2. Find all of the critical points of the function below, and classify each one (as a maximum, minimum or saddle point)

$f(x,y) = 4x + 6y - x^2 - y^2$

3. Find all of the critical points of the function below, and classify each one (as a maximum, minimum or saddle point)

$f(x,y) = x^2 + y^2 - xy^2 + 3$

Solutions:

1. For the function $f(x,y) = x^3 + 3y^2 - 4xy^2 + 4x$

a. $\dfrac{\partial f}{\partial x} = 3x^2 - 4y^2 + 4$

b. $\dfrac{\partial f}{\partial y} = 6y - 8xy$

c. $\dfrac{\partial^2 f}{\partial x^2} = 6x$

d. $\dfrac{\partial^2 f}{\partial y^2} = 6 - 8x$

e. $\dfrac{\partial^2 f}{\partial x \partial y} = -8y$

2. Find all of the critical points of the function below, and classify each one (as a maximum, minimum or saddle point)

$f(x,y) = 4x + 6y - x^2 - y^2$

$f_x = 4 - 2x$

$f_y = 6 - 2y$

$4 - 2x = 0 \qquad 6 - 2y = 0$

$4 = 2x \qquad 6 = 2y$

$x = 2 \qquad y = 3$

$f(2,3) = 4 \cdot 2 + 6 \cdot 3 - 2^2 - 3^2 = 8 + 18 - 4 - 9 = 13$

$f_{xx} = -2 \longrightarrow \cap$ max

$f_{yy} = -2$

$f_{xy} = 0$

$H = 4 \longrightarrow$ max or min

maximum at (2,3,13)

3. Find all of the critical points of the function below, and classify each one (as a maximum, minimum or saddle point)

$f(x, y) = x^2 + y^2 - xy^2 + 3$

$f_x = 2x - y^2$

$f_y = 2y - 2xy$

There are two ways to solve these. If you can do either one, and get all 3 answers, then you're doing it right.

$2x - y^2 = 0$ $2x = y^2$ $x = \frac{y^2}{2}$ (substitute in to the other equation: $2y - 2xy = 0$) $2y - 2\cdot\frac{y^2}{2}\cdot y = 0$ $2y - y^3 = 0$ $y(2 - y^2) = 0$ $y^2 = 0 \qquad 2 - y^2 = 0$ $y = 0 \qquad y^2 = 2$ $\qquad\qquad y = \pm\sqrt{2}$ (substitute back into previous: $x = \frac{y^2}{2}$) $x = \frac{0^2}{2} \qquad x = \frac{(\sqrt{2})^2}{2} \qquad x = \frac{(-\sqrt{2})^2}{2}$ $x = 0 \qquad x = 1 \qquad x = 1$ $(0, 0) \qquad (1, \sqrt{2}) \qquad (1, -\sqrt{2})$	$2y - 2xy = 0$ $2y(1 - x) = 0$ $2y = 0 \quad 1 - x = 0$ $y = 0 \quad x = 1$ (substitute into the other equation) $2x - y^2 = 0$ $2x - 0 = 0 \qquad 2\cdot 1 - y^2 = 0$ $2x = 0 \qquad y^2 = 2$ $\qquad\qquad y = \pm\sqrt{2}$ $(0, 0) \qquad (1, \sqrt{2}) \quad (1, -\sqrt{2})$

Now find $z = f(x, y)$ *and check H to determine max/min/saddle point*

$f(0,0) = 0 + 0 - 0 + 3 = 3 \qquad f(1, \sqrt{2}) = 1 + 2 - 1\cdot 2 + 3 = 4 \qquad f(1, -\sqrt{2}) = 1 + 2 - 1\cdot 2 + 3 = 4$

$f_{xx} = 2$

$f_{yy} = 2 - 2x$

$f_{xy} = -2y$

$H = 2(2 - 2x) - (-2y)^2 = 4 - 4x - 4y^2$

$H(0,0) = 4 - 0 - 0 = 4 \qquad H(1, \sqrt{2}) = 4 - 4 - 8 = -8 \qquad H(1, -\sqrt{2}) = 4 - 4 - 8 = -8$

max or min. $f_{xx} = 2$ saddle point   saddle point

min (0,0,3) $(1, \sqrt{2}, 4)$ $(1, -\sqrt{2}, 4)$