Math 156 Review and practice problems for test 1:

Skill: numerically approximate a derivative
Practice: for each of the following functions and values, choose a small variable change (h) and use it to approximate the derivative of the function at the given value.

1. $f(x) = \log(x);\ x = 3$

2. $f(t) = 2^t;\ t = 4$

Skill: use derivative rules to find the derivative of a function
Practice: find the derivative of each of these functions.

3. $f(x) = 3\sqrt{x} + \dfrac{2.5}{x^{1.6}} - \dfrac{x^3}{7} + \pi x + 10$

4. $h(x) = \dfrac{3x+7}{x^2 - 4x}$ (expand the numerator)

5. $g(x) = (3x+2)^5(5x^2 - 6x)^3$ (factor out common factors)

6. $y = \sqrt{3x^2 + 5x}$

7. $y = (\sqrt{x} + (2x+3)^4)^6$

Ones to practice after section 4.5:

8. $f(x) = x^4 \ln(x^2 + 3x)$

9. $g(x) = x^5 e^{3x+2}$

10. $h(x) = \dfrac{e^{3x} - x}{x^2 + 2}$

11. $y = \ln((2x+5)^3 + e^{5x})$

Skill: Find marginal cost, demand or profit (given a function). State the correct units. Evaluate at a given value, and interpret your answers.

12. Given the profit function in dollars for producing x clocks $P(x) = -.002x^2 + 14x - 5000$

a. Find the marginal profit function.
b. Compute the profit and the marginal profit when $x = 1000$. Tell the units for each value. Explain what the marginal profit means in this case. If the company were currently making 1000 clocks, would it be a good idea for them to increase production?
c. Compute the profit and marginal profit when $x = 4000$. Tell the units for each value. Explain what the marginal profit means in this case. If the company were currently making 4000 clocks, would it be a good idea for them to increase production?

Skill: Find the marginal and average cost, given a cost function. Know how marginal and average cost are related

13. Given the cost function: $C(x) = 3000 + 25x - .001x^2$ for producing piano benches.

a. Find the marginal cost function.
b. Find the average cost function.
c. Find the marginal and average cost when $x = 500$. Tell the units for each value, and interpret what the values mean (what do they tell you about the cost).
d. For a value of x that is larger than 500, what would you expect the relationship to be between the values of the marginal and average cost? (which would be larger? by how much?)

Skill: Find the equation of a tangent line to a function.

14. Find the equation of the tangent line to the function $y = x^2 - 3x + 1$ at the point where $x = 4$.

Answers to the review practice problems:

1. ~.145
2. ~11.1

3. $\dfrac{df}{dx} = \dfrac{3}{2}x^{-1/2} - 4x^{-2.6} - \dfrac{3}{7}x^2 + \pi$

4. $\dfrac{dh}{dx} = \dfrac{-3x^2 - 14x + 28}{(x^2 - 4x)^2}$

5. $\dfrac{dg}{dx} = \underbrace{3(3x+2)^4(5x^2-6x)^2(5(5x^2-6x)+(3x+2)(10x+6))}_{\text{this is correct, and the common factors have been factored out}} = \underbrace{3(3x+2)^4(5x^2-6x)^2(55x^2-28x-12)}_{\text{this is even more simplified}}$

6. $\dfrac{dy}{dx} = (3x^2+5x)^{-1/2}(6x+5)$

7. $\dfrac{dy}{dx} = 6(\sqrt{x} + (2x+3)^4)^5(\tfrac{1}{2}x^{-1/2} + 8(2x+3)^3)$

8. $\dfrac{df}{dx} = 4x^3\ln(x^2+3x) + \dfrac{x^4(2x+3)}{x^2+3x}$

9. $\dfrac{dg}{dx} = 5x^4e^{3x+2} + 3x^5e^{3x+2}$

10. $\dfrac{dh}{dx} = \dfrac{(3e^{3x}-1)(x^2+2) - (e^{3x}-x)2x}{(x^2+2)^2}$

11. $\dfrac{dy}{dx} = \dfrac{6(2x+5)^2 + 5e^{5x}}{(2x+5)^3 + e^{5x}}$

12. a. $P'(x) = -.004x + 14$

b. P(1000) = \$7000. The company is making a profit of \$7000 at this production level.
$P'(1000) = 10$ dollars per clock. This means the company will make approximately \$10 more in profit for each additional clock it makes.
The company will make more profit if it increases production.

c. P(4000) = \$19000. The company is making a profit of \$19,000 at this production level.
$P'(4000) = -2$ dollars per clock. This means the company will lose approximately \$20 from its profit for each additional clock it makes.
The company will make more profit if it increases production, and it will make more profit if it decreases production.

13. a. $C'(x) = 25 - .002x$

b. $\overline{C}(x) = \dfrac{3000 + 25x - .001x^2}{x}$

c. Find the marginal and average cost when $x = 100$.
$C'(100) =$ \$24.00/bench. The next bench will cost \$24.80 to produce, and the next several benches will cost about \$24.00 each to produce. When deciding whether to increase production of benches, you should consider that each additional bench will cost about \$24.00 to produce.
$\overline{C}(500) =$ \$30.50/bench. The total cost of making 500 benches, averaged out, comes to \$30.50 per bench. When pricing the piano benches, you should consider that each bench costs approximately \$30.50

d. If the number of benches increased, I would expect the average cost to remain higher than the marginal cost, but to be somewhat closer to the marginal cost than the current difference of \$6.50.

14. $y = 5x - 15$