Math 146 Test 2 practice problems solutions

1. Write an equation of a line through points (2,3) and (5,1)

You need to know the formulas: $m=\frac{y_2-y_1}{x_2-x_1}$ and $y-y_1=m(x-x_1)$

And you need to be able to simplify to a reasonable form:

$m=\frac{1-3}{5-2}=-\frac{2}{3}$ and $y-3=-\frac{2}{3}(x-2)$

Simplify in one of these ways

$$y-3=-\frac{2}{3}x+\frac{4}{3}$$
$$y=-\frac{2}{3}x+\frac{4}{3}+3$$
$$y=\frac{-2}{3}x+\frac{13}{3}$$

or

$$3(y-3)=-\frac{2}{\cancel{3}}(x-2)\cdot\cancel{3}$$
$$3y-9=-2(x-2)$$
$$3y-9=-2x+4$$
$$3y+2x=4+9$$
$$3y+2x=13$$

2. a. Write an equation of a vertical line through (1,3) $x=1$

b. Write an equation of a horizontal line through (1,3) $y=3$

3. For both parts, solve first for y to get the slope of the given line:

$$3x+2y=1$$
$$2y=-3x+1$$
$$\frac{2y}{2}=\frac{-3x}{2}+\frac{1}{2}$$
$$y=-\frac{3}{2}x+\frac{1}{2}$$

so $m=-\frac{3}{2}$

Write an equation of a line parallel to 3x+2y=1 through (1,3)

A parallel line will have slope $m=-\frac{3}{2}$ through point (1,3)

So the equation was $y-3=-\frac{3}{2}(x-1)$

and simplify:

$$y-3=-\frac{3}{2}(x-1)$$
$$y-3=-\frac{3}{2}x+\frac{3}{2}$$
$$y=-\frac{3}{2}x+\frac{3}{2}+3$$
$$y=-\frac{3}{2}x+\frac{9}{2}$$

or

$$y-3=-\frac{3}{2}(x-1)$$
$$2\cdot(y-3)=\cancel{2}\cdot\left(-\frac{3}{\cancel{2}}\right)(x-1)$$
$$2y-6=-3x+3$$
$$2y+3x=9$$

b. Write an equation of a line perpendicular to 3x+2y=1 through (1,3)

The slope perpendicular to $m=-\frac{3}{2}$ is $m=\frac{2}{3}$

The line with slope $m=\frac{2}{3}$ through (1,3) is

$$y-3=\frac{2}{3}(x-1)$$

Simplify like this:

$$y-3=\frac{2}{3}(x-1)$$
$$y-3=\frac{2}{3}x-\frac{2}{3}$$
$$y=\frac{2}{3}x-\frac{2}{3}+3$$
$$y=\frac{2}{3}x+\frac{7}{3}$$

or

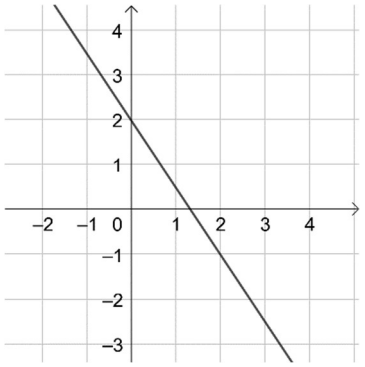
$$y-3=\frac{2}{3}(x-1)$$
$$3\cdot(y-3)=\cancel{3}\cdot\frac{2}{\cancel{3}}(x-1)$$
$$3y-9=2x-2$$
$$-2x+3y=7$$

4. Graph each of these functions or relations:

a. $3x+2y=4$

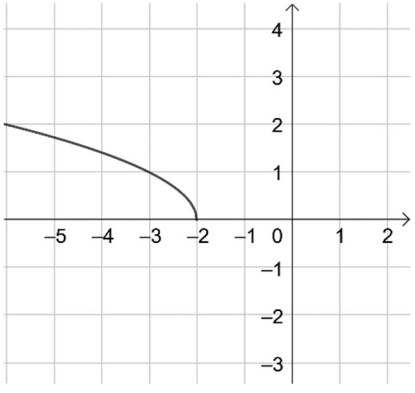
$$2y=-3x+4$$
$$\frac{2y}{2}=\frac{-3x}{2}+\frac{4}{2}$$
$$y=\frac{-3}{2}x+2$$

slope -3/2, y-intercept: 2



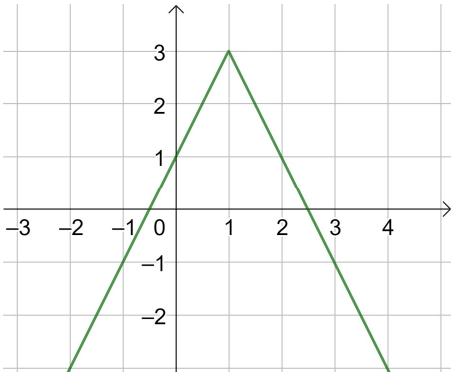
b. $y=\sqrt{-(x+2)}$

flip left-right, with center at (-2,0)



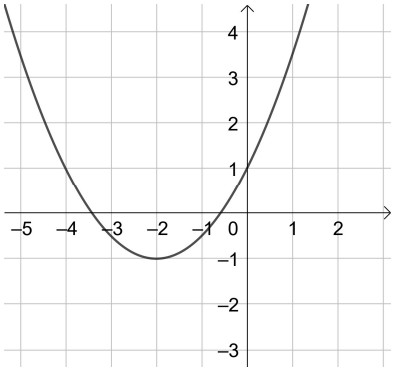
c. $y = -2\,|\,x-1\,|\,+3$ flip up-down,

stretch twice as high, center at (1,3)



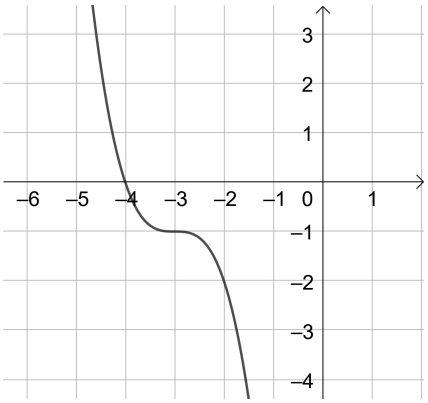
d. $y = \frac{1}{2}(x+2)^2 - 1$

flatten to height 1/2, center at (-2,-1)



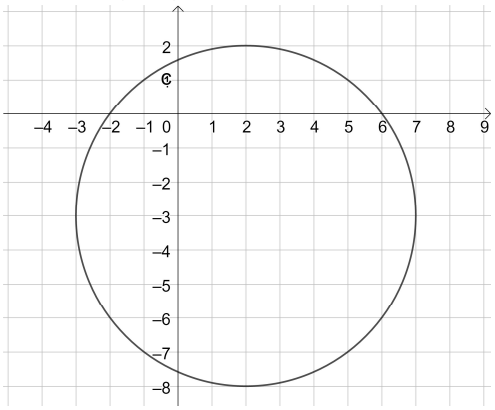
e. $y = -(x+3)^3 - 1$

flip up down, center at (-3,-1)



f. $(x-2)^2 + (y+3)^2 = 25$

center at (2,-3) radius 5



5. Graph the functions a. $y = \begin{cases} 2x-3 & \text{if} \quad x \le -2 \\ x-1 & \text{if} \quad -2 < x < 1 \\ -2x+1 & \text{if} \quad 1 \le x \end{cases}$

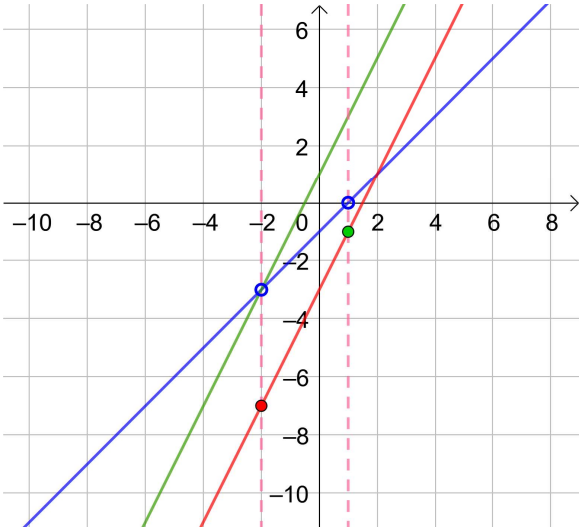
$(-2,\ 2(-2)-3) = (-2,-7)$

$(-2,\ -2-1) = (-2,-3)$

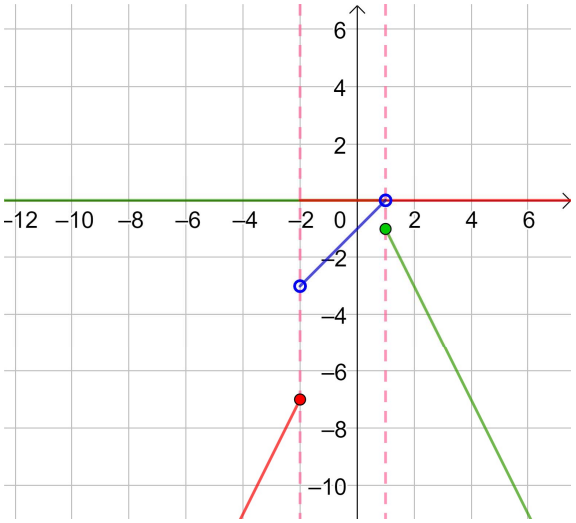
$(1,\ 1-1) = (1,0)$

$(1,\ -2\cdot 1+1) = (1,-1)$

Start with all of these graphs and some points



and then erase the parts that aren't included:



5. Graph the functions b. $y = \begin{cases} \sqrt{-x}+2 & \text{if} \quad x<0 \\ 2 & \text{if} \quad 0<x<1 \\ 2x & \text{if} \quad 1 \le x \end{cases}$

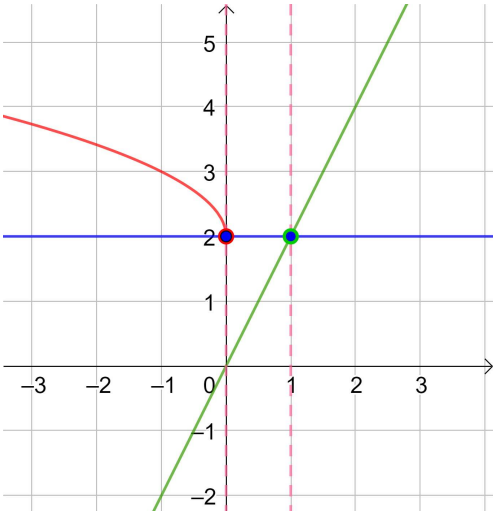
$(0,\ \sqrt{0}+2)=(0,2)$

$(0,2)$

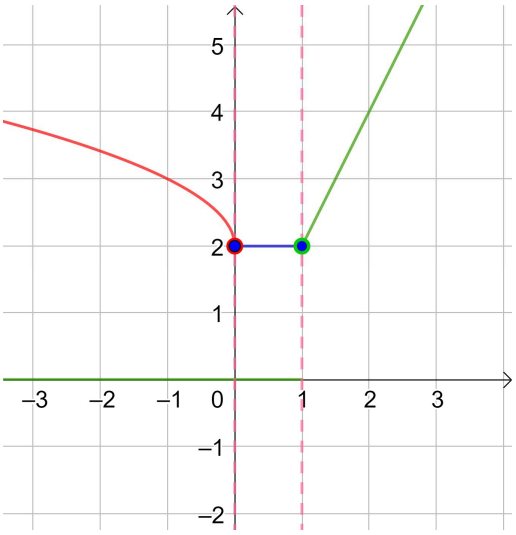
$(1,2)$

$(1,\ 2\cdot 1)=(1,2)$

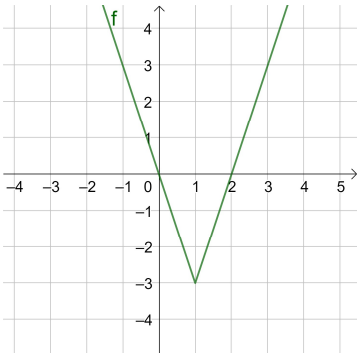
Start with all of these graphs and some points

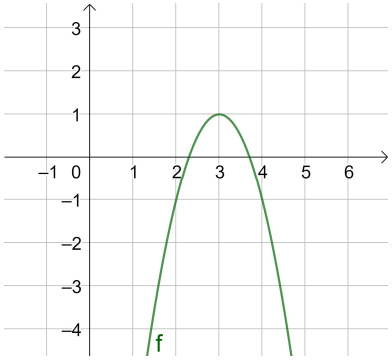


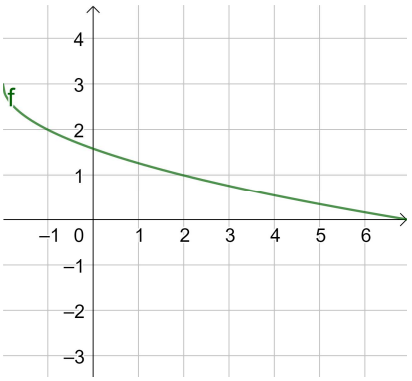
and then erase the parts that aren't included:



6. Write the equation of each of these functions or relations:



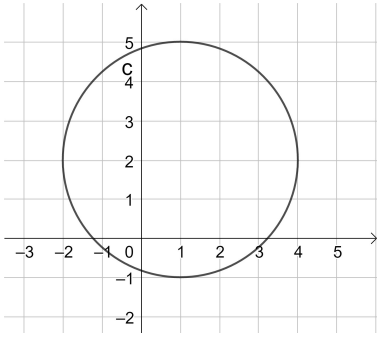


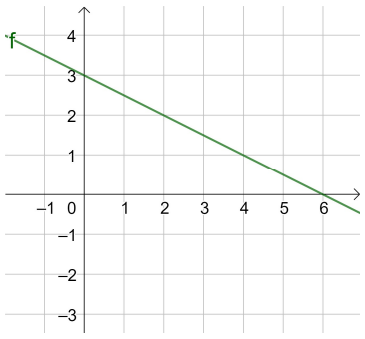


$y = 2|x-1|-3$

$y = -2(x-3)^2+1$

$y = -\sqrt{x+2}\ +3$

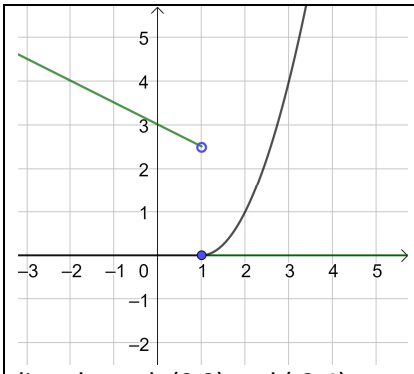




$(x-1)^2+(y-2)^2=9$

Points (6,0) and (0,3) $m = \dfrac{0-3}{6-0} = \dfrac{-1}{2}$ y-intercept: 3. Equation $y = -\dfrac{1}{2}x+3$

7. Write the equation of each of these functions:
Find the equations of the line and the parabolas, and then work on putting them together



line through (0,3) and (-2,4)

$$m=\frac{4-3}{-2-0}=-\frac{1}{2}$$

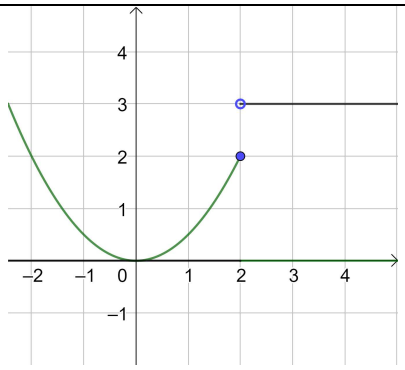
$$y=-\frac{1}{2}x+3$$

Parabola not stretched at center (1,0)

$$y=(x-1)^2$$

Line to left of x=1, parabola to the right

$$y=\begin{cases}-\frac{1}{2}x+3 & \text{if} \quad x<1\\ (x-1)^2 & \text{if} \quad x\geq 1\end{cases}$$



Horizontal line at height 3

$$y=3$$

Parabola not shifted, but squashed down by 1/2

$$y=\frac{1}{2}x^2$$

Line to the right of x=2, parabola to the left

$$y=\begin{cases}\frac{1}{2}x^2 & \text{if} \quad x\leq 2\\ 3 & \text{if} \quad 2<x\end{cases}$$

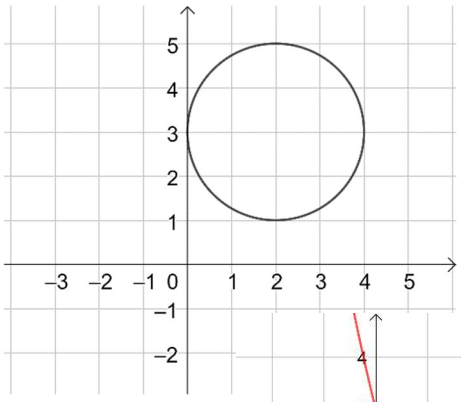
8. Put each of these equations in center-radius or vertex form by completing the square. Tell the center and radius or vertex and graph it.

a. $x^2+y^2-8x-6y+21=0$

$$x^2-8x+16+y^2-6y+9=-21+16+9$$

$$(x-4)^2+(y-3)^2=4$$

center (4,3), radius 2



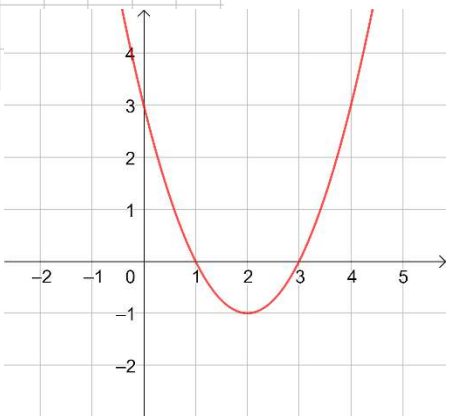
b. $y=x^2-2x+3$

$$y+4=x^2-2x+4+3$$

$$y+4=(x-2)^2+3$$

$$y=(x-2)^2-1$$

vertex (2,-1)



9. Find the vertex, axis of symmetry, x-intercepts and y-intercepts for each parabola: 3.1 # 1, 3, and 13-21 odd

1. vertex: (-3,-4)
axis of symmetry: x= -3
x-intercepts: x= -5, -1
y-intercept: 5 ($y=(0+3)^2-4=9-4=5$)

3: vertex: (-3,2)
axis of symmetry: y= -3
x-intercepts: -4,-2
y-intercept: -16 (plug x=0 into the equation)

13: vertex (2,0)
axis of symmetry: x=2
x-intercepts: x=2 solve $(x-2)^2=0$
y-intercept: 4 plug in x=0

15: vertex (-3,-4)
axis of symmetry: x= -3
solve to get x-intercepts

$$(x+3)^2-4=0$$
$$(x+3)^2=4$$
$$x+3=\pm 2$$
$$x=-3\pm 2$$
$$x=-3+2,-3-2$$
$$x=-1,-5$$

y-intercept: $(0+3)^2-4=9-4=5$

17. vertex: (-1,-3)
axis of symmetry: x = -1
solve to get x-intercepts:

$$-\frac{1}{2}(x+1)^2-3=0$$
$$-2\cdot\left(-\frac{1}{2}(x+1)^2\right)=-2\cdot 3$$
$$(x+1)^2=-6$$
$$x+1=\pm\sqrt{-6}$$

notice that the solutions will not be real numbers.
There are no x-intercepts
y-intercept: $-\frac{1}{2}(0+1)^2-3=-3\frac{1}{2}=-\frac{7}{2}$

19. $y=x^2-2x-3$ isn't factorable, so we will try to complete the square:

$$y=x^2-2x+3$$
$$y+1=x^2-2x+1+3$$
$$y+1=(x-1)^2+3$$
$$y=(x-1)^2+2$$

Vertex: (1,2)
Axis of symmetry: x=1
y-intercept: $0^2-2\cdot 0+3=3$
If we try to solve

$$x^2-2x-3=0$$

Using the quadratic formula (or using the completed square form), then we will get an imaginary part (complex number, not real number) so there are no x-intercepts.

21. $y=x^2-10x+21$ is factorable:

$$y=x^2-10x+21=(x-3)(x-7)$$

x-intercepts (3,7)
It's also pretty easy to complete the square (optional)

$$y=x^2-10x+21$$
$$y+25=x^2-10x+25+21$$
$$y+25=(x-5)^2+21$$
$$y=(x-5)^2+21-25$$
$$y=(x-5)^2-4$$

EITHER use the x-intercepts to find the axis of symmetry (half way between 3 and 7 is 5):
X = 5
And plug in to the equation to find the y-coordinate of the vertex:
$5^2-10\cdot 5+21=25-50+21=-4$ so:
Vertex: (5,-4)
OR use the completed square form to get the vertex: (5,-4)
And then the axis of symmetry: x=5
The y-intercept is $0^2-10\cdot 0+21=21$