Solutions to Sections 4.5 and 4.6 homework:

4.5 # 21. $0.05(1.15)^x = 5$

$(1.15)^x = \dfrac{5}{0.05}$ take care of any multiplication first (and simplify)

$(1.15)^x = 100$

$\log(1.15)^x = \log 100$ take the log of both sides (base 10 works great here, because we have a 100!)

$x\log(1.15) = \log(100)$ use the multiplication-exponent logarithm rule

$x = \dfrac{\log(100)}{\log(1.15)} = \dfrac{2}{\log(1.15)} = 32.950$ Divide to solve for x, and use your calculator.

23. $3\cdot(2)^{x-2} + 1 = 100$ This time there is an addition and a multiplication to take care of before using logarithms

$3\cdot(2)^{x-2} + 1 - 1 = 100 - 1$

$\dfrac{3\cdot(2)^{x-2}}{3} = \dfrac{99}{3}$

$(2)^{x-2} = 33$

$\log(2^{x-2}) = \log(33)$

$(x-2)\log(2) = \log(33)$ Don't forget the parentheses when you bring down the exponent!

$\dfrac{(x-2)\log(2)}{\log(2)} = \dfrac{\log(33)}{\log(2)}$

$x - 2 = \dfrac{\log(33)}{\log(2)}$

$x = \dfrac{\log(33)}{\log(2)} + 2 = 7.044$

25. $2(1.05)^x + 3 = 10$

$2(1.05)^x = 10 - 3$

$\dfrac{2(1.05)^x}{2} = \dfrac{7}{2}$

$(1.05)^x = \dfrac{7}{2}$

$\log(1.05)^x = \log\left(\dfrac{7}{2}\right)$

$x\log(1.05) = \log\left(\dfrac{7}{2}\right)$

$x = \dfrac{\log\left(\dfrac{7}{2}\right)}{\log(1.05)} \approx 25.677$

27. $5(1.015)^{x-1980} = 8$

$\dfrac{5(1.015)^{x-1980}}{5} = \dfrac{8}{5}$

$\log(1.015)^{x-1980} = \log\left(\dfrac{8}{5}\right)$

$\dfrac{(x-1980)\log(1.015)}{\log(1.015)} = \dfrac{\log(8/5)}{\log(1.015)}$

$x = \dfrac{\log(8/5)}{\log(1.015)} + 1980 \approx 2011.568$

47. $\log_6(2x+4)=2$

If there's only a log on one side, turn the equation into an exponent equation.

$2x+4=6^2$

$2x+4=36$

$2x=32$

$x=16$

53. $\log(x+25)=\log(x+10)+\log 4$

Combine logs (one on each side).

If there's a log on both sides, you can just remove the logs from both sides

$\log(x+25)=\log\left((x+10)\cdot 4\right)$

$x+25=4(x+10)$

$x+25=4x+40$

$-15=3x$

$-5=x$

55. $\log(x-10)-\log(x-6)=\log 2$

$\log\left(\frac{x-10}{x-6}\right)=\log 2$

$\frac{x-10}{x-6}=2$

$x-10=2(x-6)$

$x-10=2x-12$

$2=x$

61. $\log_2(x^2-100)-\log_2(x+10)=1$

$\log_2\left(\frac{x^2-100}{x+10}\right)=1$

$\frac{x^2-100}{x+10}=2^1$

$x^2-100=2(x+10)$

$x^2-100=2x+20$

$x^2-2x-120=0$

$(x-12)(x+10)=0$

So, I decided to work this problem as if I had not noticed that $x^2-100=(x-10)(x+10)$. If you noticed that, then you would have simplified much earlier, and you would have gotten to an answer by a slightly different way. This solution makes it look like there are two solutions: x=12, and x=-10. However, $\log_2(10-10)=\log_2 0$ and you can't take a log of 0 (undefined), so the only real solution is x=12, and we have to throw out -10.

4.6 # 11 for 1-c, you are just plugging in numbers for t and evaluating, so

a. $A=500e^{-0.032\cdot 4}=500e^{-0.128}\approx 439.9g.$

b. $A=500e^{-0.032\cdot 8}=500e^{-0.256}\approx 387.1g.$

c. $A=500e^{-0.032\cdot 20}=500e^{-0.64}\approx 263.6g.$

For d, we are finding the half-life (when we have half as much). You can set it up this way:

$250=500e^{-0.032\cdot t}$

Or you can set it up this way:

$0.5A_0=A_0e^{-0.032t}$

After the first division step, you should get this from both methods:

$0.5=e^{-0.032t}$

$\ln(0.5)=\ln(e^{-0.032t})$ taking ln of both sides is a little nicer than log for this problem because $\ln(e)=1$:

$\ln(0.5)=-0.032t\ln(e)$

$\frac{\ln(0.5)}{-0.032}=\frac{-0.032t}{-0.032}$

$t=\frac{\ln(0.5)}{-0.032}\approx 21.7\ yrs$

25. Compounded quarterly: $A = 60,000\left(1+\frac{.05}{4}\right)^{4\cdot 5} = 60,000(1.0125)^{20} \approx \$76922.23$

Compounded continuously $A = 60,000e^{0.0475\cdot 5} = 60,000e^{0.2375} \approx \$76084.50$

a. The higher interest rate compounded quarterly earns more money

b. The better plan earns $837.73 more over 5 years.

26. $80,000 = 60,000e^{0.0475t}$ $\frac{80,000}{60,000} = \frac{60,000e^{0.0475t}}{60,000}$ $\frac{4}{3} = e^{0.0475t}$ $\ln\left(\frac{4}{3}\right) = \ln\left(e^{0.0475t}\right)$ $\ln\left(\frac{4}{3}\right) = 0.0475t\ln(e)$ $\frac{\ln\left(\frac{4}{3}\right)}{0.0475} = \frac{0.0475t}{0.0475}$ $t = \frac{\ln(4/3)}{.0475} \approx 6.06\ yrs$ Note: $\ln(e)$ disappears because $\log_e(e) = 1$	27. The general set up would look like this (it also works if you pick a number for P and double it for 2P) $2P = Pe^{.025t}$ $\frac{2P}{P} = \frac{Pe^{.025t}}{P}$ $2 = e^{.025t}$ $\ln(2) = \ln(e^{.025t})$ $\ln 2 = .025t\ln(e)$ $\frac{\ln 2}{.025} = \frac{.025t}{.025}$ $t = \frac{\ln 2}{.025} \approx 27.7\ yrs$	29. The general set up would look like this (it also works if you pick a number for P and triple it for 3P) $3P = Pe^{.05t}$ $\frac{3P}{P} = \frac{Pe^{.05t}}{P}$ $3 = e^{.05t}$ $\ln(3) = \ln(e^{.05t})$ $\ln 3 = .05t\ln(e)$ $\frac{\ln 3}{.05} = \frac{.05t}{.05}$ $t = \frac{\ln 3}{.05} \approx 22.0\ yrs$