Section 4-2 Exponents and Exponential functions.

An exponential function is one that has a variable as an exponent:

$$y = 4^x$$

The goals for this section at this time are:

- Graph exponential functions
- Solve some equations with exponential functions

Graphing exponential functions:

To graph a basic exponential function, such as $y = 4^x$ you need to be able to evaluate it (plug in) at several numbers including: -1, 0, 1.

$4^1 = 4$

$4^0 = 1$ any number to the 0 power is 1

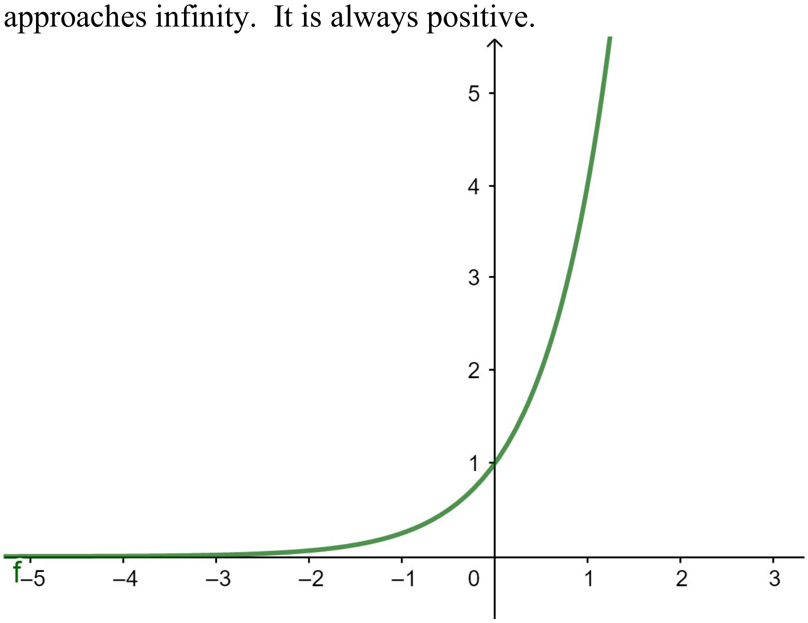
$4^{-1} = \frac{1}{4}$ this is the definition of what negative exponents do

It will help to look at a few more numbers, but in general, you're OK if you just plug in those 3.
Just this once, I'll include two more points:

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ pay attention to this example to understand negative exponents. Notice that the value is still positive. All of the outputs for $y = 4^x$ will be positive. **Negative exponents give you fractions, they don't give you negative numbers!!!**

$4^2 = 16$

Those values should help you understand the graph of $y = 4^x$: on the left it approaches 0, on the right it approaches infinity. It is always positive.



- Now, we're going to compare that to $y = \left(\frac{1}{4}\right)^x$

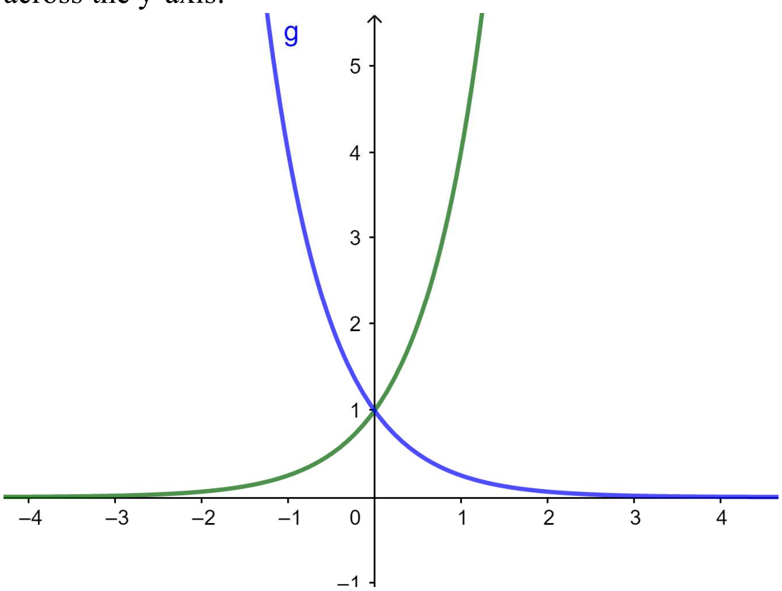
Checking the 3 basic x-values, you should get:

$$\left(\frac{1}{4}\right)^1 = \frac{1}{4}$$

$$\left(\frac{1}{4}\right)^0 = 1$$

$$\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}} = \frac{4}{1} = 4$$ notice that when you raise a fraction to a -1 power, you get the reciprocal.

If you look at these two graphs together: $y = 4^x$ in green and $y = \left(\frac{1}{4}\right)^x$ in blue, you'll notice they are reflections across the y-axis:



- Another graph to compare to: $y = 4^{-x}$

Plug in the 3 basic x-values:

$$4^{-(1)} = 4^{-1} = \frac{1}{4}$$

$$4^{-(0)} = 0$$

$$4^{-(-1)} = 4^1 = 4$$

These are the exact same y-values we got for $y = \left(\frac{1}{4}\right)^x$

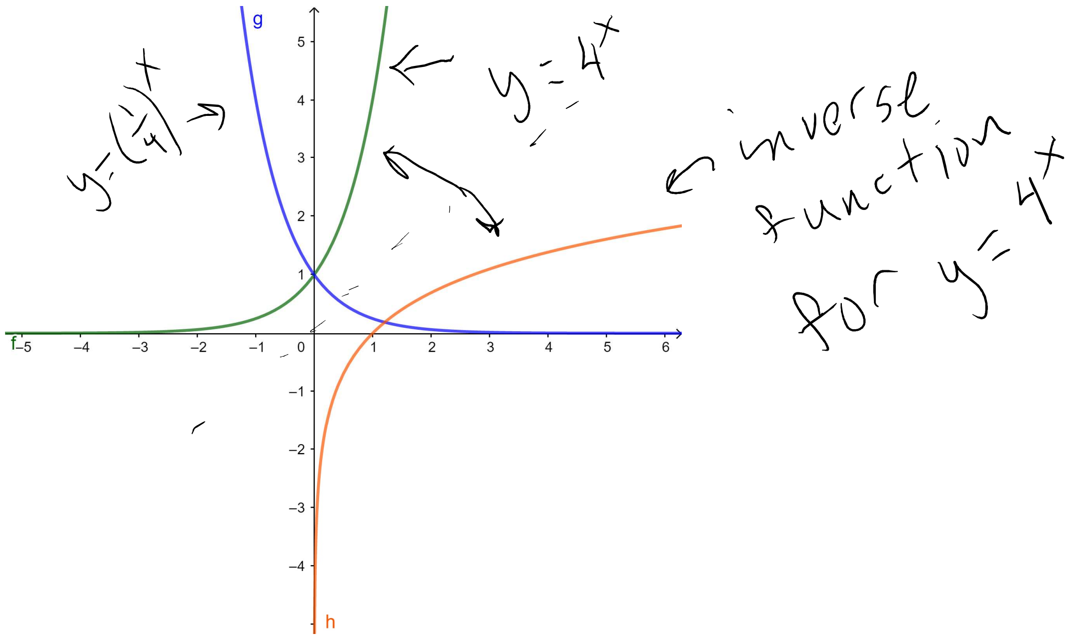
There's a reason for that! There's an exponent rule: $a^{n \cdot m} = \left(a^n\right)^m$

We can use that to say $4^{-x} = 4^{-1 \cdot x} = \left(4^{-1}\right)^x = \left(\frac{1}{4}\right)^x$

- Remember when we graphed $y = \sqrt{x}$ and $y = \sqrt{-x}$ that the graphs were left-right reflections? You can think of the graphs $y = 4^x$ and $y = 4^{-x}$ as left-right reflections because we replaced *x* with *-x*.

- Inverse functions?

You might remember that a function and its inverse are reflections, and wonder if $y = 4^x$ and $y = 4^{-x}$ are inverse functions. They are not inverse functions, and one way to see that is to look at the reflection line. $y = 4^x$ and $y = 4^{-x}$ are reflections across the y-axis. The inverse function would be a reflection across the diagonal line $y = x$. The inverse function of $y = 4^x$ is the graph shown in red:



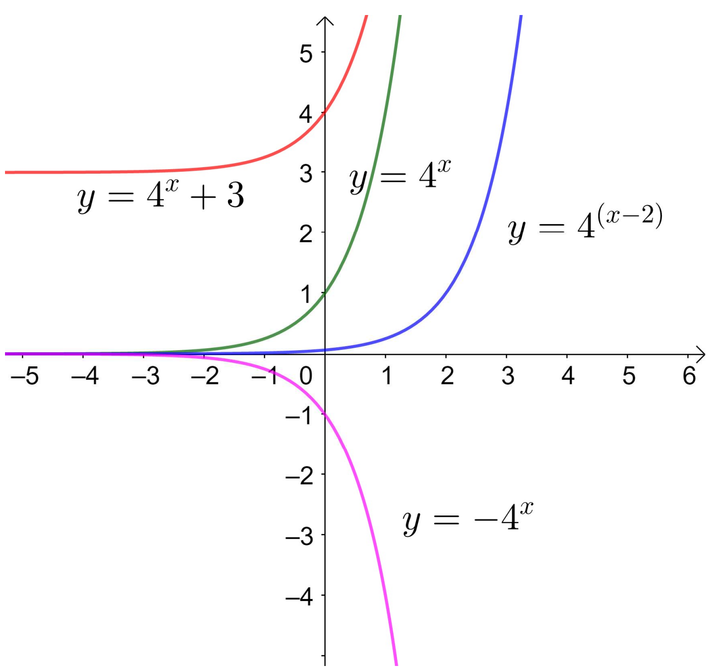
- Transforming a graph

Some of the homework problems ask you to transform graphs the way we did with other functions so:

$y = 4^{(x-2)}$ would shift left 2

$y = 4^x + 3$ would shift up 3

$y = -4^x$ means $y = (-1)\cdot 4^x$ so it reflects across the x-axis (up-down)



Solve some equations with exponential functions

There are two kinds of problems with exponents.

Ones where the variable is the base	Ones where the variable is the exponent
$x^3 = 5$	$3^x = 5$

Problems where the variable is the base

You have already learned the techniques for doing the first kind of problem in chapter 1, so this is a review:

For a simple problem (today's homework), raise both sides to the power that will cancel with the exponent you have:

$x^3 = 5$

$\left(x^3\right)^{1/3} = 5^{1/3}$

Raising an exponent to another exponent makes the exponents multiply: $\left(a^n\right)^m = a^{nm}$ so to cancel an exponent, you need to use a reciprocal exponent: if you have 3, use 1/3; if you have 3/4, use 4/3, etc.

$x^{\frac{3}{1}\cdot\frac{1}{3}} = 5^{1/3}$

$x^1 = 5^{1/3}$

$x = 5^{1/3} = \sqrt[3]{5}$

Fractional exponents are roots: $a^{1/2} = \sqrt{a}$; $a^{1/3} = \sqrt[3]{a}$. If the numerator is not 1, use a combination of roots and powers (both orders work): $a^{2/3} = \left(\sqrt[3]{a}\right)^2 = \sqrt[3]{a^2}$

Example of a problem like those in the homework (nice answers, fractional exponents):

$x^{2/3} = 9$

$\left(x^{2/3}\right)^{3/2} = 9^{3/2}$

$x^{\frac{2}{3}\cdot\frac{3}{2}} = \left(\sqrt{9}\right)^3$

$x = 3^3 = 27$

Problems where the variable is the exponent:

These problems are the big idea of this chapter: logarithms. Today's problems are warm ups where you don't need logarithms, you just need to think about exponents in an organized way. The thing that makes these doable is that you can change all of the numbers so they are exponents with the same base. For example, if you know $2^3 = 8$ and $2^2 = 4$, you can change the forms of all of these equations so that 2 is the base:

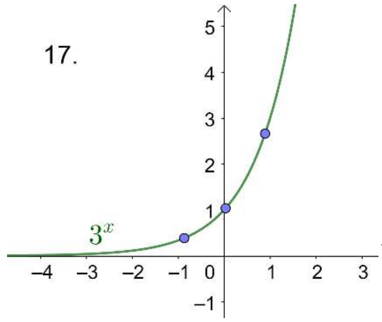
$8^x = 2$ $(2^3)^x = 2$ $2^{3x} = 2^1$	$2^{x+3} = 4$ $2^{x+3} = 2^2$	$8^{x+1} = 4^x$ $\left(2^3\right)^{x+1} = \left(2^2\right)^x$ $2^{3(x+1)} = 2^{2x}$

Once you have everything as a power of 2, you can set the exponents equal to each other to solve for x:

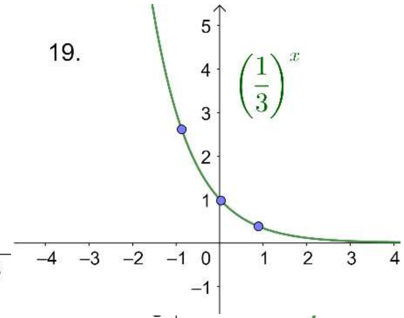
$8^x = 2$ $(2^3)^x = 2$ $2^{3x} = 2^1$ $3x = 1$ $x = \frac{1}{3}$	$2^{x+3} = 4$ $2^{x+3} = 2^2$ $x + 3 = 2$ $x = 2 - 3$ $x = -1$	$8^{x+1} = 4^x$ $\left(2^3\right)^{x+1} = \left(2^2\right)^x$ $2^{3(x+1)} = 2^{2x}$ $3(x+1) = 2x$ $3x + 3 = 2x$ $3x - 2x = -3$ $x = -3$

Solutions to homework problems:
4.2 # 17, 19, 29, 31, 33, 37, 61, 65, 66, 79, 81

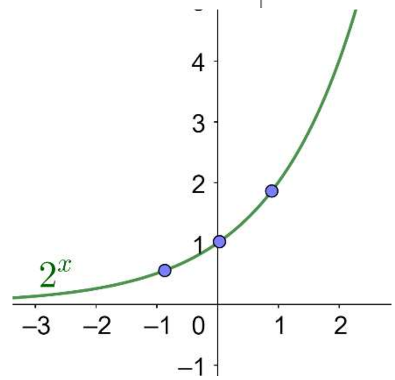
17. Graph $f(x) = 3^x$
Plot the points (-1, 1/3), (0,1) and (1,3). Make the function look like an exponential at the end.



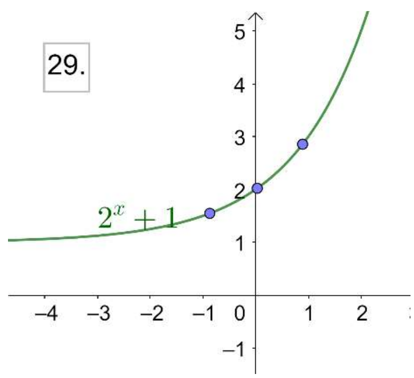
19. Graph $f(x) = \left(\frac{1}{3}\right)^x$
Plot the points (-1, 3), (0, 1) and (1,1/3). Make the function look exponential when you connect the dots.



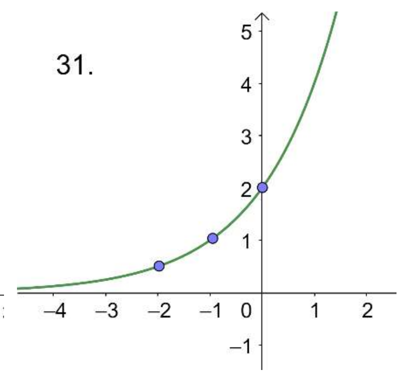
Graph $f(x) = 2^x$ Plot the points (-1,1/2), (0,0) and (1,2). Connect the dots and make it look exponential.



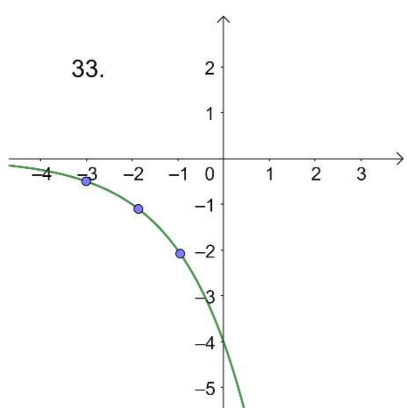
29. Graph by transforming: $f(x) = 2^x + 1$
Shift $f(x) = 2^x$ up 1. Show where the 3 basic points go.



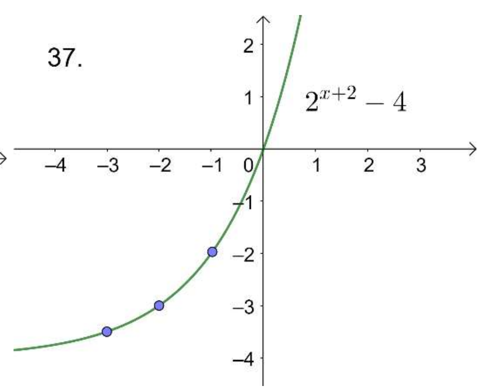
31. Graph by transforming: $f(x) = 2^{x+1}$ Shift $f(x) = 2^x$ left 1. Show where the 3 basic points go



33. Graph by transforming: -2^{x+2} Reflect $f(x) = 2^x$ up down, and shift it to the left 2. Show where the 3 basic points go.



37. Graph by transforming: $y = 2^{x+2} - 4$
Shift $f(x) = 2^x$ left 2 and down 4. Show where the 3 basic points go.



For these first 3, write the numbers as a power of a prime number if possible:

61. Solve $4^x = 2$	65. Solve $2^{3-2x} = 8$	66. Solve $5^{2+2x} = 25$
$(2^2)^x = 2$ $2^{2x} = 2^1$ $2x = 1$ $x = 1/2$ Notice that you know this because $4^{1/2} = \sqrt{4} = 2$	$2^{3-2x} = 8$ $2^{3-2x} = 2^3$ $3 - 2x = 3$ $-2x = 0$ $x = 0$	$5^{2+2x} = 5^2$ $2 + 2x = 2$ $2x = 2$ $x = 0$ (wow that was repetitively the same as 65—if I'd realized that before I would have assigned something different)

For the last two, the variable is the base number and not the exponent, so we get to use exponent rules to remove the exponent:

79. Solve: $x^{2/3} = 4$

$$\left(x^{2/3}\right)^{3/2} = 4^{3/2}$$
$$x^{\frac{2}{3}\cdot\frac{3}{2}} = 4^{3/2}$$
$$x^1 = \left(\sqrt{4}\right)^3$$
$$x = 2^3$$
$$x = 8$$

Note: you can do $4^{3/2}$ in two ways: $4^{3/2} = \left(\sqrt{4}\right)^3$ or $\sqrt{4^3}$. I think the first one is easier.

81. Solve: $x^{5/2} = 32$

$$\left(x^{5/2}\right)^{2/5} = 32^{2/5}$$
$$x^{\frac{5}{2}\cdot\frac{2}{5}} = 32^{2/5}$$
$$x^1 = \left(\sqrt[5]{32}\right)^2$$
$$x = 2^2 = 4$$

(so cool-- $32 = 2^5$ is my favorite number, so I think this is an awesome problem).