Chap. 3 review solutions

#1 possible rational zeros of

$3x^4 + \ldots + 10$

each zero would have a factor

$(\_\_ x - \_\_)$

factor of 10

factor of 3

Thinking and remembering

possible

so the zeros are

$x = \frac{\text{factor of 10}}{\text{factor of 3}}$

Factors of 10: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of 3: $\pm 1, \pm 3$

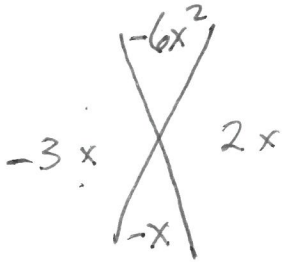
All possible zeros: $\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$

Answer

2a $2x^4 - x^3 - 3x^2 = x^2(2x^2 - x - 3)$

final answer: $x^2(2x - 3)(x + 1)$

$1 \cdot 3 = -3$ $\quad$ $-3 + 1 = -1$



	2x	-3
x	$2x^2$	$-3x$
+1	$2x$	-3

2b -3 is a zero → factor $x+3$

$$-3 \;\big|\; 1 \quad -1 \quad -11 \quad 3$$
$$\qquad\quad -3 \quad 12 \quad -3$$
$$\qquad\quad 1 \quad -4 \quad 1 \quad 0$$

$(x+3)(x^2 - 4x + 1)$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4}{2} \pm \frac{2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

Answers:

$(x+3)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$

OR

$(x+3)(x-2-\sqrt{3})(x-2+\sqrt{3})$ ← factorization

2 c. -4 is a zero, so divide!

$f(x) = 6x^3 + 11x^2 - 57x - 20$

$$\begin{array}{r|cccc} -4 & 6 & 11 & -57 & -20 \\ & & -24 & +52 & 20 \\ \hline & 6 & -13 & -5 & 0 \end{array}$$

Factorization so far:

$$(x+4)(6x^2 - 13x - 5)$$

↓ factor this

Guess & check

$(6x \quad)(x \quad 5)$

$(3x - 5)(2x + 1)$

$(3x + 1)(2x - 5)$ ∴

OR

$-30x^2$

$-10x$ $\quad -3x$

$-15x \quad 2x$

$-13x$

	$3x$	1
$2x$	$6x^2$	$2x$
-5	$-15x$	-5

$3x+1$

$2x-5$

Complete factorization:

$$\boxed{(x+4)(3x+1)(2x-5)}$$

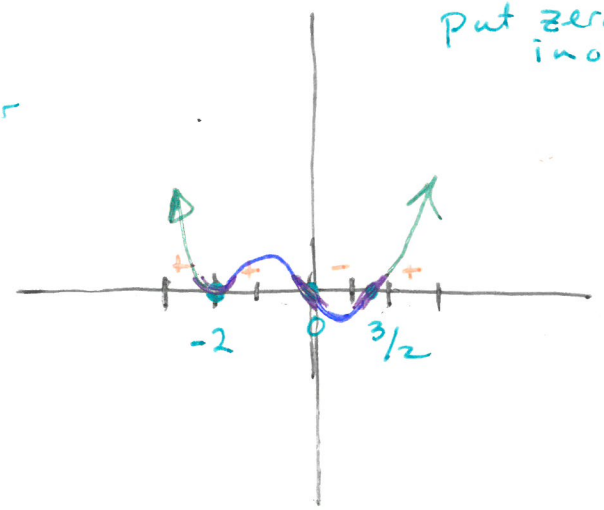
3a. $y = x(x+2)^2(2x-3)$

zeros: 0, −2, 3/2

multiplicity: 1, 2, 1

> do first

do second: put zeros in order



Sign chart ← do third

	−2	0	3/2	← zeros in order
x	− (−1)	−	+ (1)	+
$(x+2)^2$	+ $(-3+2)^2$	+	+ $(1+2)^2$	+
$2x-3$	−	−	− $2\cdot1-3$	+ $2\cdot2-3$
	+	+	−	+

↑ all factors with exponents: $(x+2)^{2}$

see video for more sign chart instructions

do 4th, put ± on graph

do 5th figure out each zero and draw one of these

(check your sign chart multiplicity by comparing  1 or 3

 multiplicity 2)

6th connect with smooth curves

7th End behavior:

$y \approx x(x)^2(2x) = 2x^4$ 

put this end behavior on graph

(check w/ sign chart

+ − − − +)

3b.

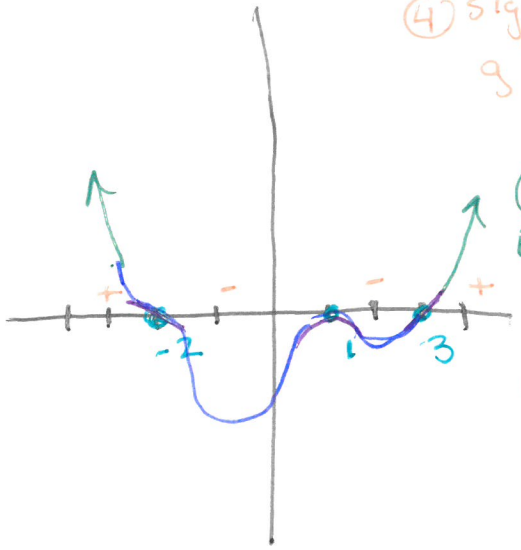
$$y = (x-1)^2(x-3)^3(x+2)$$

① Zeros & multip.

Zeros	1	3	−2
multip.	2	3	1

② zeros in order

④ signs on graph



⑧ End behavior on graph.

③ sign chart

	-2	1	3
$(x-1)^2$	+	+	+
$(x-3)^3$	−	−	−
$(x+2)$	−	+	+
	+	−	−

$(2-3)^3$ $(4-3)^3$

$-3+2$ $0+2$

← zeros in order

$(\ )^2$ is all +

Check 2 numbers for each factor

↑ factors w/ exponents (multiplicity)

Draw graph near zeros

Connect between zeros

⑦ End behavior:

$$y \approx (x)^2(x)^3(x) = x^6$$

Note end behavior possibilities:

x^2, x^4, x^6 $-x^2, -x^4, -x^6$

x, x^3, x^5, x^7 $-x, -x^3, -x^5, -x^7$

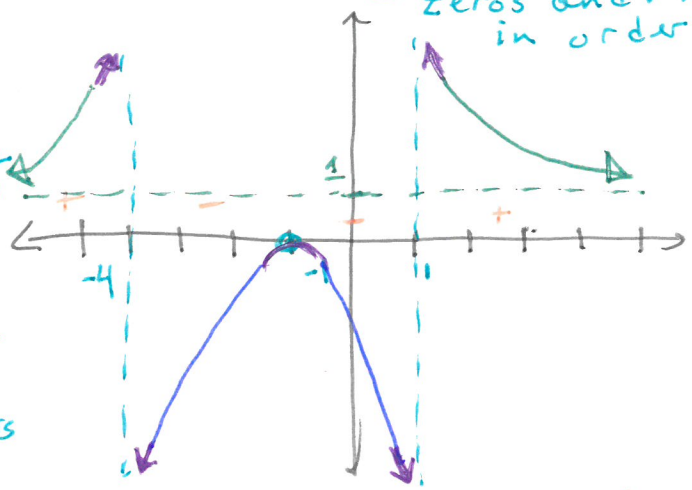
The sign chart should help. too.

3c. $y = \dfrac{(x+1)^2}{(x-1)(x+4)}$

① → zeros $x=-1$ (mult. of 2)

Vertical asymptotes $x=1, x=-4$ (mult of 1 for both)

② Zeros and V-A in order



③ sign chart

all zeros and V-A in order

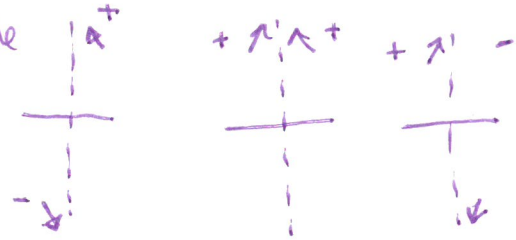
	−4		−1		1	
$(x+1)^2$	+	+	+	+	+	all + b/c ()^2
$(x-1)$	−	−	0−1 −	2−1 +		check 2 numbers
$(x+4)$	−5+4 −	+	0+4 +	+		
	+		−	−	+	

all factors w/ powers

④ = on graph

⑤ Draw near zeros and V-asymptotes

for example



⑥ Connect between zeros and asymptotes w/ smooth curves

⑦ End behavior

$y \underset{\nearrow}{\approx} \dfrac{(x)^2}{(x)(x)} = \dfrac{x^2}{x} = 1$

approx, when $x \to \infty$

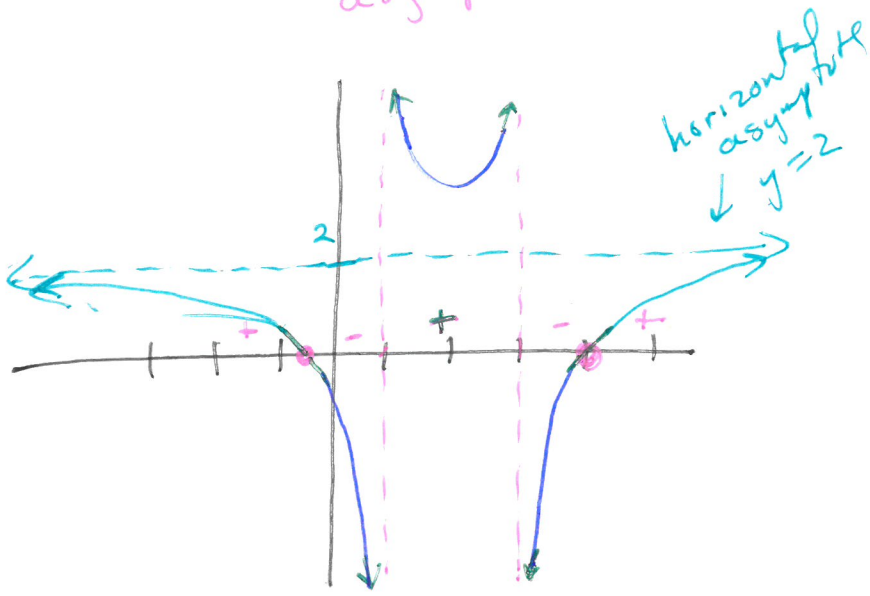
horizontal asymptote $y=1$

⑧ Draw End behavior. Connect to the pieces you already have graphed

3d. $y = \dfrac{(2x+1)(x-4)}{(x-1)(x-3)}$ → $x = -\frac{1}{2}, 4$ zeros

→ $x = 1, 3$ vertical asymptotes

	$-\frac{1}{2}$	1	3	4
$(2x+1)$	$2(-\frac{1}{2})+1$	$0+1$	+	+
$(x-4)$	−	−	−	$3-4$ −	$5-4$ +
$(x-1)$	−	−	+	+
$(x-3)$	−	−	−	+
	+	−	+	−



End behavior: $\dfrac{(2(100)+1)(100-4)}{(100-1)(100-3)} \approx \dfrac{200 \cdot 100}{100 \cdot 100}$

How to get end behavior →

$$\frac{(2x+1)(x-4)}{(x-1)(x-3)} \approx \frac{2x \cdot x}{x \cdot x} = \frac{2x^2}{x^2} = 2$$

3e

$$y = \frac{x}{(x-3)(x+2)}$$

① zeros: $x = 0$ (mult. 1)

V-asymp: $x = 3, -2$ (both mult. 1)

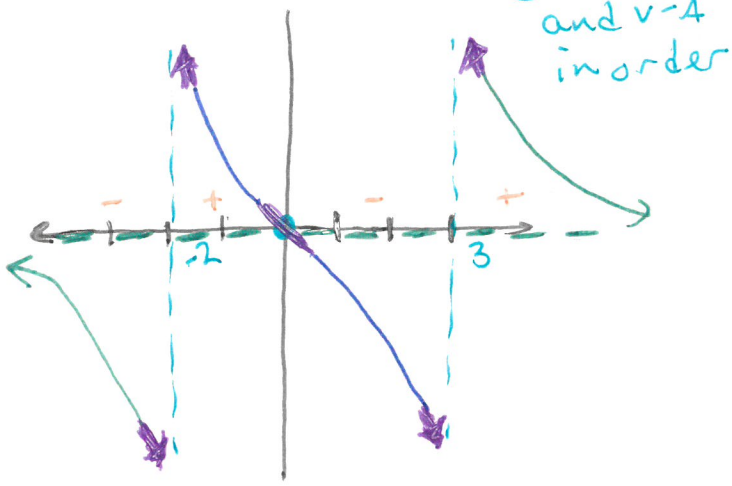
③ zeros and asymptotes in order

	−2		0		3	
		−1		+1		
x	−	−	0	+	+	
$x-3$	−	−	−3	−	0	+
$x+2$	0	+	+2	+	+5	+
	−	+	−	+		

factors

plug in 2 numbers for each row

② zeros and v-A in order



④ +, − on graph

⑤ Draw near zeros and V-asymptotes

⑥ Connect between zeros and V-As

⑦ End behavior:

$$y \approx \frac{x}{(x)(x)} = \frac{x}{x^2} = \frac{1}{x}$$

when $x \approx 100$ or $x \approx -100$

$y \approx 0$

horizontal asymptote

$y = 0$

⑧ put on horizontal asymptotes and connect to the rest of the graph

Math 146 Review for Half-Exam on Chapter 3:

1. List all of the possible rational zeros of a polynomial, such as $f(x)=3x^4-2x^3+7x+10$

2. Completely factor a polynomial, given some information about some of the roots:

a. $f(x)=2x^4-x^3-3x^2$ $= x^2(2x^2-x-3)$

b. $f(x)=x^3-x^2-11x+3$ given -3 is a zero.

c. $f(x)=6x^3+11x^2-57x-20$ given -4 is a zero

3. Graph each of these functions. Include your work showing how you figured out:

- Zeros
- Vertical asymptotes (if any)
- A sign chart
- End behavior

a. $y=x(x+2)^2(2x-3)$

b. $y=(x-1)^2(x-3)^3(x+2)$

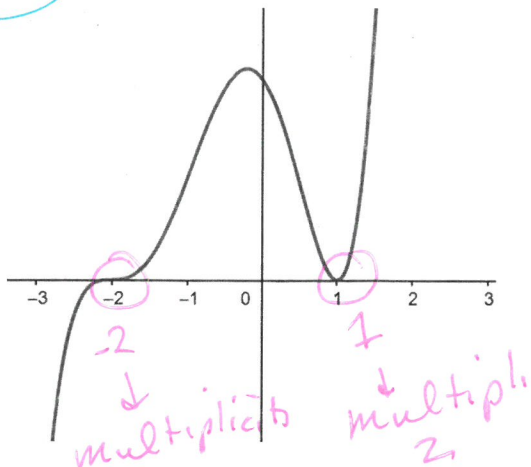
c. $y=\dfrac{(x+1)^2}{(x-1)(x+4)}$

d. $y=\dfrac{(2x+1)(x-4)}{(x-1)(x-3)}$

e. $y=\dfrac{x}{(x-3)(x+2)}$

4. Write an equation of a function that will have the graph behavior shown (it should agree on zeros and signs):

a.

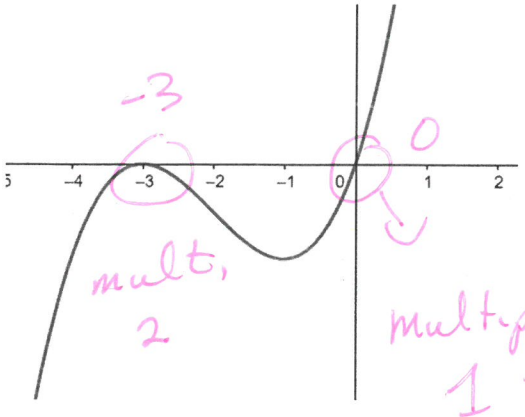


-2 multiplicity 3

1 multiplicity 2

Ans $(x+2)^3(x-1)^2$

b.

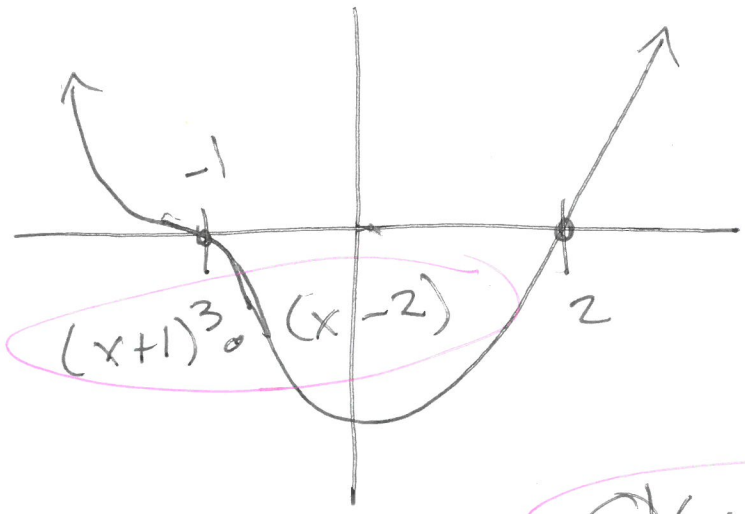


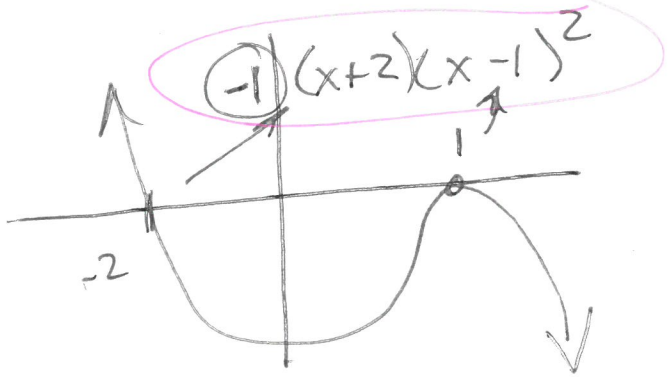
-3 mult. 2

0 multiplicity 1

Ans: $(x+3)^2 \cdot x$ ($x+0$) or ($x-0$) also OK

Extra In-class examples
More eg like #4





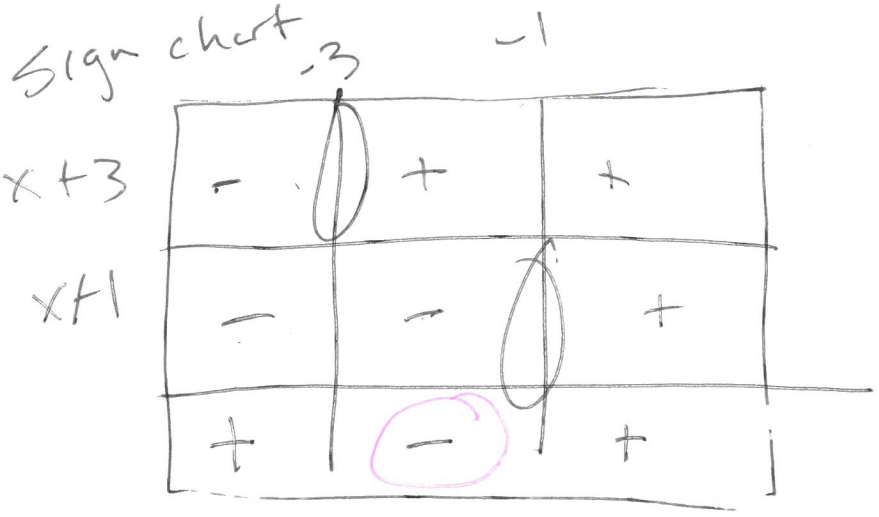
Retest from test 1

May redo

#

$(x+3)(x+1) < 0$

Sign chart



(9)

Interval: $(-3, -1)$

(8)

$2x + 5 \geq 2$

$2x \geq -3$

$x \geq -\frac{3}{2}$ $\quad [-3/2, \infty)$

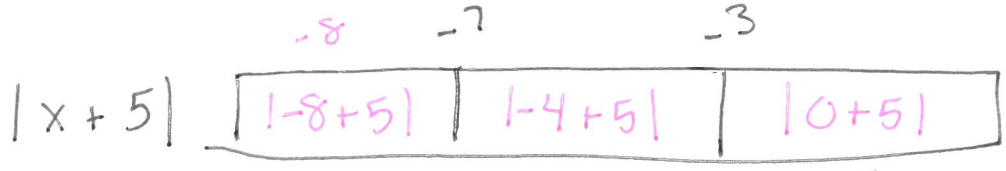
$|x + 5| < 2$

(11) (or 14)

$|x + 5| = 2$

$x + 5 = 2$ $\qquad$ $x + 5 = -2$

$x = -3$ $\qquad$ $x = -7$



Answer interval: $(-7, -3)$