Section 2.8: Function composition

If you have two functions, you can **compose** them, which means plugging one function into the other.

First thing: we use function notation to show what to plug in to a function. Here are some examples:

If $f(x) = x^2 - 3x + 1$ then

$f(2) = 2^2 - 3 \cdot 2 + 1$

$f(-2) = (-2)^2 - 3(-2) + 1$ don't forget parentheses when it's a negative number!

$f(a) = a^2 - 3a + 1$ it doesn't have to be a number you're plugging in

$f(a+1) = (a+1)^2 - 3(a+1) + 1$ you need parentheses here too!

$f\left(\frac{1}{x+1}\right) = \left(\frac{1}{x+1}\right)^2 - 3\left(\frac{1}{x+1}\right) + 1$ it can get pretty complicated with what you plug in

Function composition us just one of these complicated ones.
The notation $f \circ g(x)$ means $f(g(x))$, which means to plug $g(x)$ in to $f(x)$

Example 1: $f(x) = x^2 - 3x + 1$ and $g(x) = \frac{1}{x+1}$, then

$$f \circ g(x) = f(g(x)) = f\left(\frac{1}{x+1}\right) = \left(\frac{1}{x+1}\right)^2 - 3\left(\frac{1}{x+1}\right) + 1$$

Notice that I replaced x in $f(x)$ by the definition of $g(x)$.

Also notice that I replaced all of the x's in $f(x)$ by $g(x) = \frac{1}{x+1}$

Example 2: we can use any names for the functions, and the composition of the functions works the same way. For example, with functions:

$h(x) = \sqrt{x+1}$ and $k(x) = x^2 + 4x$

$$h \circ k(x) = h(k(x)) = h(x^2 + 4x) = \sqrt{(x^2 + 4x) + 1} = \sqrt{x^2 + 4x + 1}$$

and we can also do:

$$k \circ h(x) = k\left(h(x)\right) = k(\sqrt{x+1})\left(= \sqrt{x+1}\right)^2 + 4\sqrt{x+1} + 1$$

Solutions to the homework problems on the next page.

65. a. $f\circ g(x)=f(g(x))=f(x+3)=\sqrt{x+3}$
The numbers you are allowed to plug in (domain) are the ones where you're not taking the square root of a negative number, so you need to solve:
$x+3\geq 0$ so $x\geq -3$ and the domain is $[-3,\infty)$
b. $g\circ f(x)=g(f(x))=g(\sqrt{x})=\sqrt{x}+3$
The domain is $[0,\infty)$

67. a. $f\circ g(x)=f(g(x))=f(x^2+3x-1)=\left(x^2+3x-1\right)^3$, which has domain $(-\infty,\infty)$
b. $g\circ f(x)=g(f(x))=g(x^3)=\left(x^3\right)^2+3x^3-1$, which also has domain $(-\infty,\infty)$

71. a. $f\circ g(x)=f(g(x))=f(x+1)=\dfrac{2}{x+1}$
The domain (allowed numbers for x) is everything that does not have 0 in the denominator, so everywhere except $x=-1$. In interval notation, the domain is $(-\infty,-1)\cup(-1,\infty)$
$g\circ f(x)=g(f(x))=g\left(\dfrac{2}{x}\right)=\dfrac{2}{x}+1$
The domain (allowed numbers for x) is everything that does not have 0 in the denominator, so everywhere except $x=0$. In interval notation, the domain is $(-\infty,0)\cup(0,\infty)$