Compound Interest

total at end
↓
$$A = P\left(1+\frac{r}{n}\right)^{tn}$$

$P = 1000$

$r = 5\% = .05$

$t = 3$ years

$n \to$ compounded per year

$$A = 1000\left(1+\frac{.05}{n}\right)^{n\cdot 3}$$

$$= 1000\left(\left(1+\frac{.05}{n}\right)^{n}\right)^{3}$$

$$= 1000\left(\left(1+\frac{1}{m}\right)^{m}\right)^{.05\cdot 3}$$

$m = f(r, t)$

→ a number not infinity

$m \to \infty$

give it a name: e !

$$= 1000\, e^{.05\cdot 3}$$

$$= 1000\, e^{r\cdot t}$$

Compounded continuously: $A = Pe^{r\cdot t}$

$$A = 1000\left(e^{(.05)\cdot 3}\right)$$

1161.83

Ex 1. $\log(x \cdot y \cdot z) = \log(x) + \log(y) + \log(z)$

$\log(x+1) + \log(x-1) = \log\left((x+1)(x-1)\right)$

Ex 2

$\log\left(\frac{2}{x}\right) = \log 2 - \log x$

$\log(x+1) - \log(3) = \log\left(\frac{x+1}{3}\right)$

Ex 3

$\log\left(\frac{x\,y}{2\,z}\right) = \log(x) + \log(y) - \log(2) - \log z)$

$\log(x) - \log(x+1) + \log(x-1) - \log(5)$

$= \log\left(\frac{x(x-1)}{(x+1)5}\right)$

$\log(x \cdot y) = \log x + \log y$

$\log(x+y) =$ ---- nothing to do (trick problem!)

Expand each of theses

$\log_3 \frac{4p}{q} =$

$\log_5 \frac{5\sqrt{7}}{3} =$

$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} =$

$\log_3 \frac{4p}{q} = + \log_3 4 + \log_3 p - \log_3 q$

$\log_5 \frac{5\sqrt{7}}{3} = \log_5 5 + \log_5 \sqrt{7} - \log_5 3$

$= \log_5 5 + \log_5 7^{1/2} - \log_5 3$

$= \boxed{\log_5 5} + \frac{1}{2} \log_5 7 - \log_5 3$

$1 + \frac{1}{2} \log_5 7 - \log_5 3$

$$\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_p \left(\frac{m^5 n^4}{t^2}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left(\log_p\left(\frac{(m^5)(n^4)}{(t^2)}\right)\right)$$

$$= \frac{1}{3}\left(\log_p(m^5) + \log_p(n^4) - \log_p(t^2)\right)$$

$$= \frac{1}{3}\left(5\log_p m + 4\log_p n - 2\log_p(t)\right)$$

$$= \frac{5}{3}\log_p(m) + \frac{4}{3}\log_p(n) - \frac{2}{3}\log_p(t)$$

① $\log_a x - \log_a y - \log_a m$

$$= \log_a \left(\frac{x}{ym}\right)$$

② $5\log_3 x + \frac{1}{2}\log_3 y - \frac{1}{3}\log_3 z^6$

$\log_3 x^5 + \log_3 \sqrt{y} - \log_3 (z^6)^{1/3}$

z^2

$$\log_3 \left(\frac{x^5 \sqrt{y}}{z^2}\right)$$

4.2 # 875, 89, 91

4.3 # 63, 67, 69, 71, 73, 77, 79, 81

4.4 # 11, 15, 17, 19, 23, 27, 45, 49, 51

$3 \log 2 = \log(2^3)$

Present Value

$r = .04$ 10 years

I want 30,000 at the end (future value)

How much do I need to invest? ← present value

Quarterly

$A = P\left(1 + \frac{.04}{4}\right)^{4 \cdot 10}$

$30{,}000 = P\underbrace{\left(1 + \frac{.04}{4}\right)^{4 \cdot 10}}_{L}$

$\frac{30{,}000}{1.4888} = \frac{P(1.4888)}{1.4888}$

$20{,}150.45 = P$

Continuously

$A = Pe^{rt}$

$30{,}000 = P\boxed{e^{.04 \cdot 10}}$

$30{,}000 = P(1.4918)$

$\frac{30{,}000}{1.4918} = P$

$20{,}109.93 = P$

88. $P = 56{,}780$

5.3% ← yearly interest rate

$\frac{.053}{4}$ ← quarter interest rate

↖ interest each quarter

for 5 years:

$$(56{,}780)\left(1+\frac{.053}{4}\right)^{5\cdot 4}$$

for 23 quarters!

$$(56{,}780)\left(1+\frac{.053}{4}\right)^{23}$$