Final exam   Thurs   12/19/19
10:15 – 12:15   here

4.5 #47 $\log_3 [(x+5)(x-3)] = 2$

$$3^2 = (x+5)(x-3)$$

$$\underset{-9}{9} = x^2 - 3x + 5x - 15 \underset{-9}{}$$

$$0 = x^2 + 2x - 24$$

$$0 = (x - 4)(x + 6)$$

$x^2 + 6x - 4x - 24$ ☺

$x = 4$, $x = -6$

double check: $\log_3[9 \cdot 1] = 2$ ✓ $\log_3(-1 \cdot (-9)) = \log(9) = 2$

23. $3(2)^{x-2} + 1 = 100$

$$\phantom{3(2)^{x-2}+1=}\underset{-1}{} \quad \underset{-1}{}$$

$$\frac{3(2)^{x-2}}{3} = \frac{99}{3}$$

$$(2)^{x-2} = 33$$

$$\log(2)^{x-2} = 33$$

$$\frac{(x-2)\log 2}{\log 2} = \frac{\log 33}{\log 2}$$

$$x-2 = \frac{\log 33}{\log 2}$$

$$x = \frac{\log 33}{\log 2} + 2$$

$$x \approx 5.0444 + 2$$
$$x \approx 7.0444$$

$$\log_2(33) = x-2$$

$$\frac{\log(33)}{\log(2)} = x-2$$

$$x-2 = 5.0444$$
$$x = 5.0444 + 2$$
$$x = 7.0444$$

61. $\log_2(x^2 - 100) - \log_2(x+10) = 1$

$\log_2\left(\dfrac{x^2 - 100}{x+10}\right) = 1$

$\dfrac{x^2 - 100}{x+10} = 2^1$

$x^2 - 100 = 2(x+10)$

$x^2 - 100 = 2x + 20$

$x^2 - 2x - 120 = 0$

$(x-12)(x+10) = 0$

$x = 12$ $x = -10$

$\underset{\nearrow 12^2}{}$ 12

$\log_2(144 - 100) - \log_2(12+10)$

$= \log_2(44) - \log_2(22)$

$= \log_2\left(\dfrac{44}{22}\right) = \log_2(2) = 1$

$\log_2((-10)^2 - 100) - \log_2(-10+10)$

$\log_2(0) - \log_2(0)$

$\uparrow$ not defined

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55. $\log(x-10) - \log(x-6) = \log 2$

$\log\left(\dfrac{x-10}{x-6}\right) = \log 2$

$\dfrac{x-10}{x-6} = 2$

$x - 10 = 2(x-6)$

$x - 10 = 2x - 12$

$\underset{-x \ +12}{} \quad \underset{-x \ +12}{}$

$12 - 10 = x$

$2 = x$

$\emptyset$

no solutions

?

$\log(2-10) - \log(2-6) = \log(2)$

$\log(-8) - \log(-4)$

$\uparrow \qquad \uparrow$

not defined

Formulas you should know:

$$\log_b n + \log_b m = \log_b(n \cdot m)$$

$$\log_b n - \log_b m = \log_b\left(\frac{n}{m}\right)$$

$$\log_b n^k = k \log_b n$$

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

point-slope form

$$\text{slope} \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \begin{array}{l} y = m(x - x_1) + y_1 \\ y - y_1 = m(x - x_1) \end{array}$$
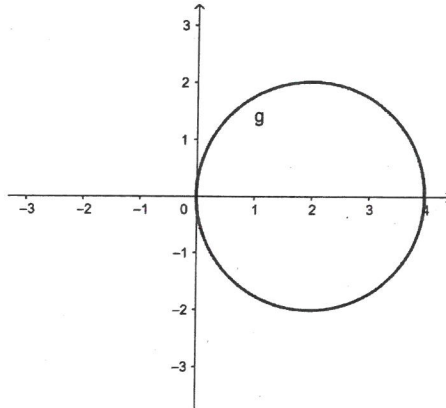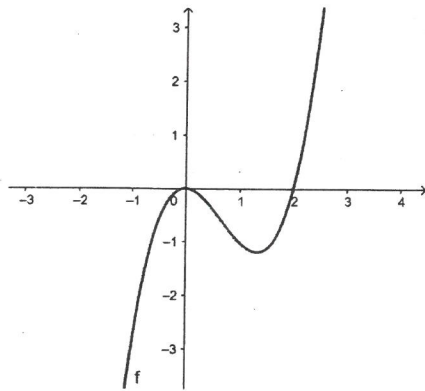
Practice problems from Chapter 4 (about half of the final exam, maybe a little more):

1. For each of these relations, tell whether it is a function or not:

a. $\{(2,3),(2,4),(3,5)\}$ b. $\{(2,3),(3,3),(4,3)\}$

(is y a function of x: x is the input, y is the output)

c. $y = x^2$ d. $x = y^2$ e. $x = y^3$

f.   g.

2. For each of these pairs of functions, find $f \circ g(x)$. Simplify, and use the simplified form to tell whether the functions are inverses or not:

a. $f(x) = 3x + 5$ $g(x) = \frac{1}{3}x - 5$ b. $f(x) = \frac{1}{x-2}$ $g(x) = \frac{1}{x} + 2$

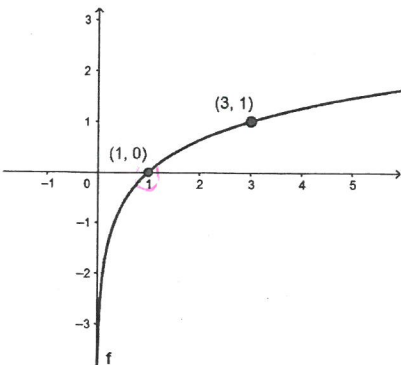3. Find $f \circ g(x)$ for $f(x) = x^2 + 3x + 1$ $g(x) = x - 2$

4. Find the inverse function for each of these functions:

a. $f(x) = 3x + 4$ b. $f(x) = 2x^3 + 1$ c. $f(x) = 3 \cdot 2^x$

5. If the graph of $y = f(x)$ given below, graph:

a. $y = f^{-1}(x)$ b. $y = f(x) + 1$ c. $y = f(x+2)$

(show exactly what happens to the two points labelled)

$f^{-1}(x)$ means
inverse function
of f

The interest rate equations are:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$ for interest compounded n times per year and

$$A = Pe^{rt}$$ for interest compounded continuously.

6. If an amount of $3,000 is invested at an annual interest rate of 2.8% compounded <u>quarterly</u>, what is the value of the investment after 5 years? (Round to the nearest penny)

7. If the same amount ($3,000) is invested at the same annual interest rate (2.8%), but the interest is compounded <u>continuously</u>, what is the value of the investment after 5 years? (Round to the nearest penny)

8. How much money must be invested at an annual interest rate of 2.8%, compounded <u>quarterly</u>, so that the investment will yield $6000 in 8 years? (Round to the nearest penny)

9. How much money must be invested at an annual interest rate of 2.8%, compounded <u>continuously</u>, so that the investment will yield $6000 in 8 years? (Round to the nearest penny)

10. If an amount of money (can you pick any amount for this problem?) is invested at an annual interest rate of 2.8%, compounded <u>quarterly</u>, how long will it take for the investment to double in value? (Round to the nearest hundredth of a year)

11. If an amount of money (yes—you can pick any amount for this problem!) is invested at an annual interest rate of 2.8%, compounded <u>continuously</u>, how long will it take for the investment to double in value? (Round to the nearest hundredth of a year)

12. Plutonium-241 decays according to the function $A(t) = A_0 e^{-0.053t}$. How long will it take some amount of Plutonium (it doesn't matter how much!) to decay to half of its amount? (Round to the nearest tenth of a year)

13. Expand these logarithmic expressions:

a. $\log_3\left(\dfrac{3\sqrt{x}}{y^2}\right)$ b. $\log\left(\dfrac{x}{\sqrt{yz}}\right)$ c. $\ln\left(3\sqrt[4]{xy}\right)$

14. Combine these logarithmic expressions:

a. $\log x + 2\log y - 3\log z - \log w$ b. $\ln(3) + 2\ln x - \ln(x+1)$ c. $\log_2 x + 2\log_2(x+1)$

15. Solve the exponential equations:

a. $2^{(x-1)} = 8$ b. $3(2^{x+1}) = 12$ c. $5(1.2)^{x-4} = 14$ d. $4 \cdot 2^{3x} = 18$

(Round to 4 decimal places)

16. Solve the logarithmic equations:

a. $\log_3(2x+1) - \log_3(x-3) = 2$ b. $\ln(x+1) + \ln(x+2) - \ln(4) = \ln(3)$

c. $\log_2(2x+5) - \log_2(x-1) = 3$ d. $\log(x+1) + \log(x-2) = 1$

e. $\log(x+1) - \log(x-2) = \log(4)$

(Round to 4 decimal places)

Review practice problems from chapters 1-3 (about half of the final exam, maybe a little less)

Math 146 Final Exam review items from chapters 1-3

Some problems like these from Chapter 1 will be on the final exam:

17. Solve by factoring: $2x^2 - 5x - 12 = 0$ ←

18. Solve and leave the answer in exact simplified form (square roots, fractions, but no decimals)

a. $2x^2 - 2x + 3 = 0$ b. $x^2 + 4x - 8 = 0$

19. Factor each expression:

a. $x^2 + 4x$ b. $x^3 - 10x^2 + 24x$ *factor out x first*

20. Solve each equation:

a. $\dfrac{x+1}{x^2-9} - \dfrac{x+3}{x^2+5x+6} = \dfrac{4}{x^2-x-6}$ b. $(x+3)^{2/3} = 36$

21. An ideal gas satisfies the equation PV=nRT, where P is the pressure in atm, V is the volume in Liters, T is the temperature in degrees kelvin, n is the number of moles, and R is a constant.

a. Solve for the constant R.

b. Given n=2.5 mol of a gas, when the temperature is 275 K and the pressure is .95 atm, then the volume is 59 L. If the air pressure stays the same (.95 atm), and the amount of gas stays the same (2.5 mol) and the temperature increases to 310 K, what is the new volume? (Round to the nearest tenth of a liter)

There will be some problems like these from Chapter 2:

22. Graph the functions a. $y = \begin{cases} 2x+3 & \text{if} & x \le -2 \\ x^2 & \text{if} & -2 < x < 1 \\ -2x+5 & \text{if} & 1 \le x \end{cases}$ b. $y = \begin{cases} \sqrt{-x}+2 & \text{if} & x < 0 \\ 2 & \text{if} & 0 \le x < 1 \\ 2x & \text{if} & 1 \le x \end{cases}$

23. Write the equation of each of these functions or relations:
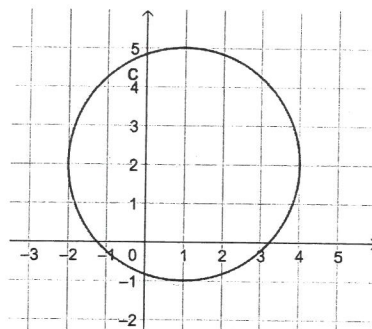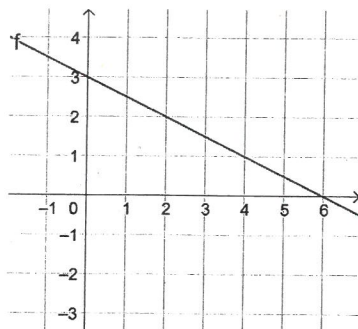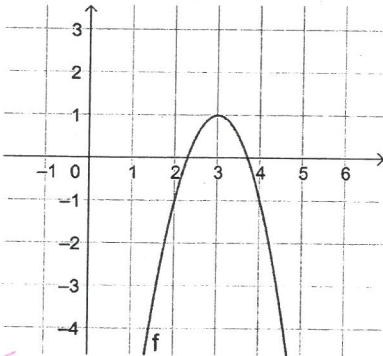
x², lines, circle

24. Write an equation of a line through points (2,3) and (5,1). Give the answer in either general or slope-intercept form

There will be a factorization problem like one of these from Chapter 3:

25. Completely factor each of these polynomials:

a. $f(x) = 2x^3 + 13x^2 + 17x - 12$ given that (x+4) is a factor

b. $f(x) = 3x^3 + 4x^2 - 17x - 6$ given that 2 is a zero.