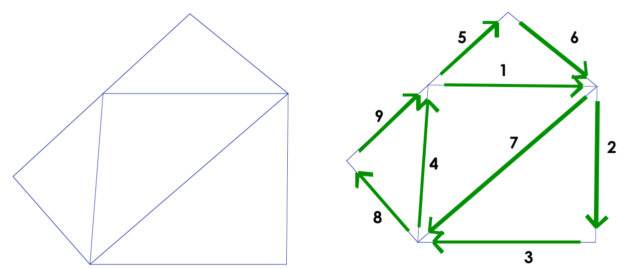
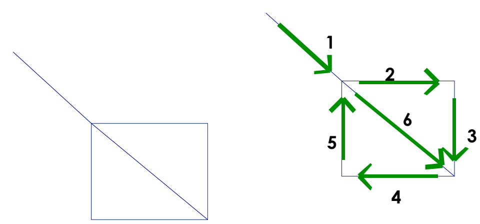
Traceable graphs (Euler circuits and paths)

We're going to work on tracing graphs. These aren't x-y graphs from algebra, these are just collections of vertices and edges. The same rules hold for these edges as for the ones we used for Euler Characteristic calculations:

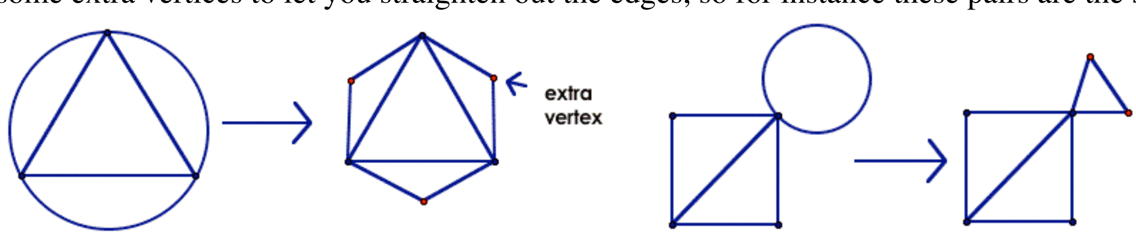
- Vertices have to go at crossings and at endpoints. You are also allowed to put in extra vertices to split an edge into two edges.
- Edges go between veritices. They are squiggly and sometimes zig-zaggy lines. Edges always start and end at vertices (though sometimes they can start and end at the same vertex

Part 1: A *traceable* graph is one where you can trace the whole thing, without ever going over the same line twice, and without lifting your pen/pencil. Here is an example of a traceable graph.

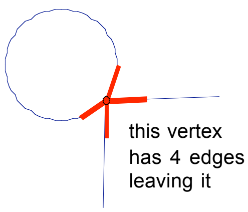
This path is an ***Euler circuit*** because it starts and ends at the same vertex.	
This path is an ***Euler path***, but not an Euler circuit because it doesn't start and end in the same place	

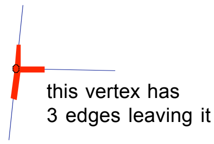
Do part 1 of the assignment now. You can check your work by using the applet at http://www.math.okstate.edu/%7Ewrightd/1493/euler/index.html (the applet will show an Euler circuit or path if there is one, but if there is one, there are usually a lot of solutions, so yours doesn't have to be the same to be right)

If you want to use the applet to figure out something with curved edges, you'll have to put in some extra vertices to let you straighten out the edges, so for instance these pairs are the same:

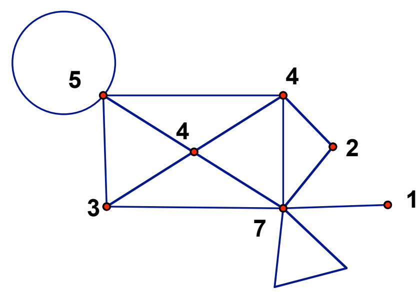


Part 2: We're going to analyze the vertices in a graph, by giving each one a ***valence number***. The valence number of a vertex is the number of lines coming out of it (kind of like when we counted the number of faces or edges meeting at a vertex for the polyhedra). If an edge starts and ends at the same vertex, you count it twice.





Here is an example:

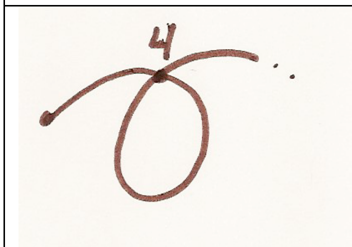
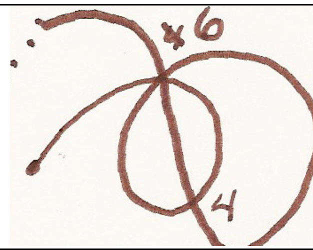
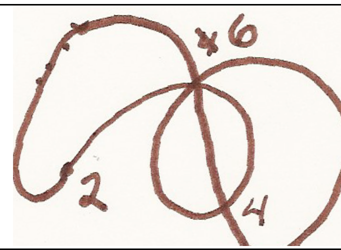
This graph has 4 vertices with odd valences (5, 3, 7, and 1) This graph has 4 vertices with even valences (4, 4, and 2) It would also be right if you put in 2 more vertices of valence 2 at the corners of the triangle. (this graph cannot be traced)	

Do Part 2 of the assignment now

You can find more info at
http://www.ctl.ua.edu/math103/euler/3importa.htm#Euler%20Circuits:%203%20Important%20Questions

More Explanations: Euler circuit

Try this: Put your pencil on the paper and draw a doodle by starting your pencil at a dot and going around and around making crossings, but not lifting your pencil until you get back to your starting point.

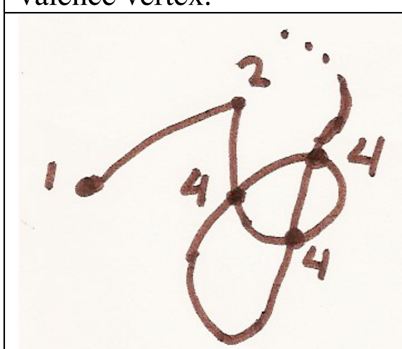
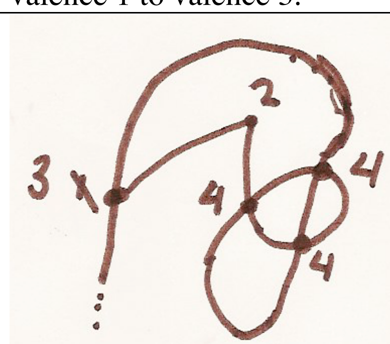
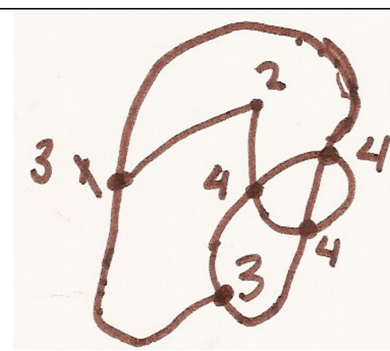
Notice that each time your doodle crosses itself, you get a vertex with valence 4:	If you cross a vertex that already has valence 4, you get a vertex with valence 6.	Keep going until you get back to the point where you started:
		

This has to be an Euler circuit because that's the way you drew it (you never took your pen off the paper), and you always have vertices that have an even valence, because crossings make the valance go up by 2.

The rule for Euler circuits is: *If you have a connected edge-vertex graph, and all of the valences are even, then you can find an Euler circuit that works for that graph.*

Euler path

Try this: Put your pencil on the paper and draw a doodle by starting your pencil at a dot and going around and around making crossings, but not lifting your pencil until you get back to your starting point.

Again, each time you cross the graph, you make an even valence vertex.	If you cross a the dot you started with, it changes from valence 1 to valence 3.	End your doodle in a different place from where you started:
		

This has to be an Euler path because that's the way you drew it, and you always have vertices that have an even valence except for the first and the last point, and those are always odd

The rule for Euler paths is. If you have a connected edge-vertex graph, and all of the valences are even except two, then you can find an Euler path that works for that graph, and *the two vertices that are odd will always be the first and the last one in the path.*

If there are more than two odd vertices, it can't be traced without lifting your pen/pencil, because there would have to be more than two first and last points.